

On Smarandache M-Semigroup

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Abstract

In this research, we defined the term a smarandache M – semigroup (S-M-semigroup) and studied some basic properties.

Also defined smarandache fuzzy M-semigroup and some elementary properties about this concepts are discussed .

Introduction

In 1965 Zadeh introduced the concept of fuzzy set, in 1971 Rosenfeld formulated the term of fuzzy subgroup. In 1994 W.X.Gu , S.Y.Li and D.G.Chen studied fuzzy groups and gave some new concepts as M- fuzzy groups . In 1999 W.B.Vasantha introduced the concepts of smarandache semigroups . Smarandache fuzzy semigroups are studied in 2003 by W.B.Vasantha .

In this research , the concept of Smarandache M- fuzzy semigroup are given and its some elementary properties are discussed

1- Preliminaries

Definition(1.1) : A fuzzy set μ of a group G is called a fuzzy subgroup if

$$\mu(xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \} \text{ for every } x,y \in G . [2]$$

Definition(1.2): A fuzzy subgroup μ of a group G is called a fuzzy normal subgroup if

$$\mu(xyx^{-1}) \geq \mu(y) \text{ for every } x, y \in G. [2]$$

Definition(1.3) : A group with operators is an algebraic system consisting of a group G , set M and a function defined in the product $M \times G$ and having value in G such that, if ma denotes the elements in G determined by the element m of M , then

$$m(ab) = (ma)(mb) \text{ hold for all } a, b \text{ in } G, m \text{ in } M. [4]$$

We shall usually use the phrase "G is an M-group" to a group with operators.

Definition (1.4): If μ is a fuzzy set of G and $t \in [0,1]$ then $\mu_t = \{x \in G \mid \mu(x) \geq t\}$ is called a t-level set μ . [1]

Definition (1.5): Let G and G' both be M -groups, f be a homomorphism from G onto G' , if $f(mx) = mf(x)$ for every $m \in M, x \in X$, then f is called a M -homomorphism. [4]

Definition (1.6): Let G be M -group and μ be a fuzzy subgroup of G if $\mu(mx) \geq \mu(x)$ for every $x \in G, m \in M$, then μ is said to be a fuzzy subgroup with operators of G , we use the phrase μ is an M -fuzzy subgroup of G instead of a fuzzy subgroup with operators of G . [4]

Proposition (1.7): If μ is an M -fuzzy subgroup of G , then the following statements hold for every $x, y \in G, m \in M$: [4]

- 1- $\mu(m(xy)) \geq \mu(mx) \wedge \mu(my)$
- 2- $\mu(mx^{-1}) \geq \mu(x)$

Proposition (1.8): Let G and G' both M -groups and f an M -homomorphism from G onto G' , if μ' is an M -fuzzy subgroup of G' then $f^{-1}(\mu')$ is an M -fuzzy subgroup of G . [4]

Proposition (1.9): Let G and G' both M -groups and f an M -homomorphism from G onto G' if μ is an M -fuzzy subgroup of G then $f(\mu)$ is an M -fuzzy subgroup of G' . [1]

Definition (1.10): Let S be a semigroup, S is said to be a smarandache semigroup (S -semigroup) if S has a proper subset P such that P is a group under the operation of G . [2]

Definition (1.11): Let S be an S -semigroup. A fuzzy subset $\mu : S \rightarrow [0,1]$ is said to be smarandache fuzzy semigroup (S -fuzzy semigroup) if μ restricted to at least one subset P of S which is a subgroup is a fuzzy subgroup. [3]

that is for all $x, y \in P \subset S, \mu(xy^{-1}) \geq \min \{ \mu(x), \mu(y) \}$.

this S- fuzzy semigroup is denoted by $\mu_p: P \rightarrow [0,1]$ is fuzzy group .

Definition (1.12): A semigroup H with operators is an algebraic system consisting of a semigroup H , set M , and a function defined in the product $M \times H$ and having values in H such that, if ma denotes the element in H determined by the element a in H and the element m in M , then $m(ab) = (ma)(mb)$, $a, b \in H$ and $m \in M$ then H is M – semigroup . [4]

Definition (1.13): Let f be a function from a set X to a set Y while μ is fuzzy set of X then the image $f(\mu)$ of μ is the fuzzy set $f(\mu) : Y \rightarrow [0,1]$ defined by : [1]

$$f(\mu(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if } f^{-1}(y) = \phi \end{cases}$$

Definition (1.14): Let f be a function from a set X to a set Y while μ is fuzzy set of Y then the inverse image $f^{-1}(\mu)$ of μ under f is the fuzzy set $f^{-1}(\mu) : X \rightarrow [0,1]$ defined by $f^{-1}(\mu)(x) = \mu(f(x))$. [1]

2-The Main Results

In this section we shall define smarandache M – semigroup and smarandache fuzzy M -semigroup and given some its results .

Definition (2.1): Let H be M - semigroup . H is said to be a smarandache M – semigroup (S-M-semigroup) if H has a proper subset K such that K is M - group under the operation of H .

Definition (2.2): Let H be a S - M – semigroup. A fuzzy subset $\mu : H \rightarrow [0,1]$ is said to be smarandache fuzzy M -semigroup if μ restricted to at least one subset K of H which is subgroup is fuzzy subgroup .

Definition (2.3): Let H be a S-M- semigroup . A fuzzy subset $\mu : H \rightarrow [0,1]$ is said to be smarandache fuzzy M -semi group if restricted to at least one subset K of H which is M - subgroup is fuzzy M - subgroup

Definition (2.4): Let S and S' be any two S- semigroup . A map ϕ from S to S' is said to be S- semigroup homomorphism if ϕ restricted to a subgroup $A \subset S \rightarrow A' \subset S'$ is a group homomorphism .

Definition (2.5): Let H and H' be any two S - M - semigroup . A map φ from H to H' is said to be S - M - semigroup homomorphism if φ restricted to a M - subgroup

$A \subset S \rightarrow A' \subset S'$ is M - homomorphism .

Proposition (2.6): If μ is S - fuzzy M -semigroup of S - M - semigroup then :

- 1) $\mu_K(m(xy)) \geq \min \{ \mu_K(mx) , \mu_K(my) \}$
- 2) $\mu_K(mx^{-1}) \geq \mu_K(x)$

For all $x \in M$, $x, y \in K$

Proof : μ is S - fuzzy M -semigroup

Then there exist subset K of H which is M - subgroup such μ restricted of K which is fuzzy M - subgroup

i.e . $\mu_K: K \rightarrow [0,1]$, M - fuzzy subgroup

for all $x, y \in K$, $m \in M$, it is clear that

- 1) $\mu_K(m(xy)) \geq \mu_K((mx)(my))$
 $\geq \min \{ \mu_K(mx) , \mu_K(my) \}$
- 2) $\mu_K(mx^{-1}) = \mu_K(mx)^{-1}$
 $\geq \mu_K(mx)$
 $\geq \mu_K(mx)$ ■

Proposition (2.7): Let G be S - semigroup , μ fuzzy set of G , then μ is an S - fuzzy M - semigroup

of G iff $\forall t \in [0,1]$, μ_t is an S - M - semigroup $\mu_t \neq \emptyset$.

Proof : It is clear μ_t is semigroup of G while $\mu_t \neq \emptyset$ holds .

for any $x \in \mu_t$, $m \in M$

$$\mu(mx) \geq \mu(x) \geq t$$

hence mx in μ_t , hence μ_t is an M - semigroup of G .

since S -fuzzy M - semigroup $\exists K \subset G$ subgroup $\exists \mu_t: K \rightarrow [0,1]$

fuzzy M - subgroup.

$$\mu_{K_t} = \{ x \in K \mid \mu_K(x) \geq t \} .$$

It is clear μ_{K_t} is group .

hence μ_t S-M- semigroup .

Conversely ,

Since μ_t S-M- semigroup then there exists a proper subset K of G such that K is M-subgroup .

If there exists $x \in K$, $m \in M$ such that $\mu_K(mx) < \mu_K(x)$.

$$\text{let } t = \frac{1}{2} (\mu_K(mx) + \mu_K(x))$$

then $\mu_K(x) > t > \mu_K(mx)$

$mx \notin \mu_{K_t}$ so here emerges a contradiction .

$\mu_K(mx) \geq \mu_K(x)$ always holds for any $x \in K, m \in M$.

μ_K is M- fuzzy subgroup

hence μ is S – fuzzy M- subgroup . ■

Proposition(2.8): Let H and H' both be S-M- semigroup and f as S-M- semigroup homomorphism from H onto H' . if μ' is an S- fuzzy M- semigroup

of H' then $f^{-1}(\mu')$ is an S- fuzzy M-semigroup of H .

Proof:

Since $f : H \rightarrow H'$ is as S-M- semigroup homomorphism then f restricted to M- subgroup .

$A \subset S \rightarrow A' \subset S'$ is M- homomorphism ,

$f^{-1}(\mu')_A : A \rightarrow [0,1]$ such that A M-subgroup ,

For any $m \in M$, $x \in A$

$$\begin{aligned} f^{-1}(\mu')_A(mx) &= \mu'_A(f(mx)) \\ &= \mu'_A m(f(x)) \geq \mu'_A(f(x)) \\ &= f^{-1}(\mu')(x) \end{aligned}$$

$f^{-1}(\mu')$ is S- fuzzy M- semigroup ■

Proposition(2.9): Let H and H' both be S-M- semigroups and f as S-M- semigroup homomorphism from H onto H' . if μ is an S- fuzzy M- semigroup of H then $f(\mu)$ is an S- fuzzy M- semigroup of H' .

Proof:

Since $f : H \rightarrow H'$ is as S-M- semigroup homomorphism then f restricted to M- subgroup .

$A \subset S \rightarrow A' \subset S'$ is M- homomorphism

$f(\mu)_{A'} : A' \rightarrow [0,1]$ such that A' M-subgroup ,

For any $m \in M, y \in A'$

$$\begin{aligned} f(\mu)(my) &= \sup \mu(x) , x \in f^{-1}(my) \\ &= \sup \mu(x) , f(x)=my \\ &\geq \sup \mu(mx') , f(mx')=mx , mx' \in H \\ &= \sup \mu(x') , mf(x')=my , mx' \in H \\ &\geq \sup \mu(x') , f(x')=y , x' \in H \\ &= f(\mu)(y) \end{aligned}$$

hence $f^{-1}(\mu')$ is S- fuzzy M- semigroup ■

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