Interpolation by Using Bézier Curve Numerically with Image Processing Applications

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Abstract

In this paper we find new method to interpolation by using Bézier method numerically. The problem studied here is the way of determine the control points. We have a new method in this field, named by proposed method, and explain the possibility of applying the proposed method, the estimation of block concealment points of image are studied.

الاندراج باستخدام منحني بزَير عدديا مع تطبيقات المعالجة الصورية

مصطفى عباس فاضل الشمري
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الخلاصة

في هذا البحث وجدنا طريقة جديدة للاندراج بأستخدام طريقة بَزير عددياً. مشكلة الدراسة هنا تكمن في أمكانية تحديد نقاط السيطرة. وقد حصلنا على طريقة جديدة في هذا المجال سميت بالطريقة المقترحة، و تم توضيح أمكانية تطبيق الطريقة المقترحة عن طريق ايجاد حشد النقاط الضائعة في الصورة كما جاء في الدراسة.

1- Introduction

Bézier method depends on a control points rather than an interpolation points because control points are better to use than points on the curve for stability reasons, but what happens when you are constructing the curve from a sampling method or scanning method. We get points on the model, and want to construct a model from the given points. We are not given the control points, but must construct them. Rather than artistically trying to match the curve by placing control points, it is useful to automatically generate the control points to get an approximate curve and then to weak the control points as desired. Classical numerical methods such as Lagrange, least square, and spline are given a unique numerical function with a certain ratio of error. This default error becomes a problem in many branches of science such as the automobile industry.

In this paper, the calculation of control points of Bézier Curves forms are studied. The proposed method was applied in image processing problems.

2- Basic Definitions and Theorems :

In this section, the basic Definitions and theorems with properties are presented as follows:

Definition (2-1)[1] : A function $\boldsymbol{y = f(x)}$ is given only at discrete points such as $(\boldsymbol{x_0, y_0}), (\boldsymbol{x_1, y_1}), \ldots, (\boldsymbol{x_{n-1}, y_{n-1}}), (\boldsymbol{x_n, y_n})$. To find the value of '$\boldsymbol{y}$' at any other value of '$\boldsymbol{x}$' there exist a continuous function $\boldsymbol{f(x)}$ may be used to represent the $\boldsymbol{n+1}$ data values with $\boldsymbol{f(x)}$ passing through the $\boldsymbol{n+1}$ nodes. Then one can find the value of $\boldsymbol{y}$ at any given value of $\boldsymbol{x}$. This is called interpolation [2].

Definition (2-2) (Lagrange Polynomials)[3,4]

For a given set of $n+1$ nodes x_i , the Lagrange polynomials are the $n+1$ polynomials ℓ_i define by

$$\ell_i(x_j) = \begin{cases} 0 & if \quad i \neq j \\ 1 & if \quad i = j \end{cases} \quad \ldots(\mathbf{1})$$

Then we define the interpolating polynomial as

$$P_n(x) = \sum_{i=0}^{n} y_i \ell_i(x) \qquad \ldots\ldots(2) \quad , \quad \text{where } y_i \text{ are interpolation points}$$

If each Lagrange polynomial is of degree at most n, then P_n also has this property. The basis can be characterized as follows:

$$\ell_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j} \quad \dots\dots(3)$$

Definition (2-3) [1]: Bernstein polynomials of degree n are defined by

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} \qquad \text{for } i = 0, 1, 2, \dots, n,$$

$$\text{where } \binom{n}{i} = \frac{n!}{i!\,(n-i)!} \quad \dots\dots(4)$$

In general, there are $n + 1$ Bernstein polynomials of degree n.

Definition (2-4) [5]: Given a set of control points $\{\boldsymbol{P}_i\}_{i=0}^{n}$, where $\boldsymbol{P}_i = (x_i, y_i)$, a ***Bézier curve of degree*** *n* is $\mathbf{P}(t) = \sum_{i=0}^{n} P_i B_i^n(t)$ where $B_i^n(t), for\ \ i = 0, \dots, n$, are the Bernstein polynomials of degree n, and $t \in [0,1]$.

3-Proposed Method for Finding Control Points :

We can find Bézier curve passing approximately through four or more interpolation points. Fig. 3.1 shows this method.

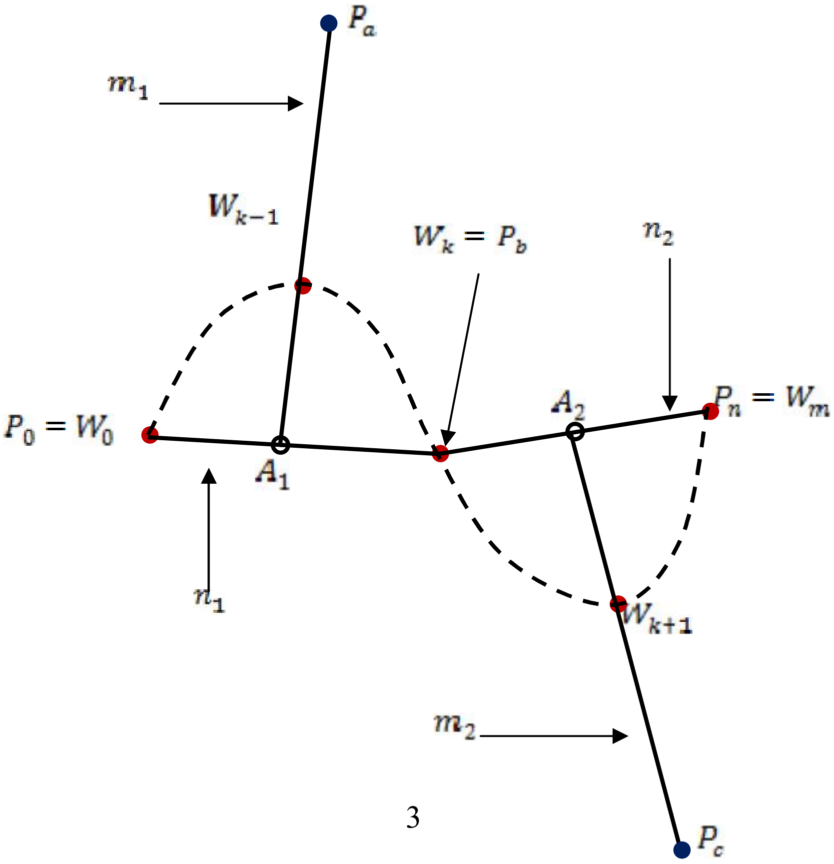


Fig. 3.1 Proposed Method

where:

$W_0 = (Wx_0, Wy_0), W_{k-1} = (Wx_{k-1}, Wy_{k-1}),$

$W_k = (Wx_k, Wy_k), W_{k+1} = (Wx_{k+1}, Wy_{k+1}), W_m = (Wx_m, Wy_m)$

A_1 is a midpoint between W_0 and W_k.

A_2 is a midpoint between W_k and W_m.

W_{k-1} is a maximal length of $\mathcal{L}(\overline{A_1, W_i})$, $i = 0,1,\ldots,k$.

W_{k+1} is a maximal length of $\mathcal{L}(\overline{A_2, W_j})$, $j = k, k+1, \ldots, m$.

n_1 is a straight line where $W_0 \in n_1$ and $W_k \in n_1$.

n_2 is a straight line where $W_k \in n_2$ and $W_m \in n_2$.

m_1 is a straight line where $A_1 \in m_1$ and $W_{k-1} \in m_1$.

m_2 is a straight line where $A_2 \in m_2$ and $W_{k+1} \in m_2$.

P_a is a control point where $\mathcal{L}(\overline{P_a, W_{k-1}}) = \frac{5}{3}\mathcal{L}(\overline{W_{k-1}, A_1})$.

P_b is a control point where $P_b = W_k$.

P_c is a control point where $\mathcal{L}(\overline{P_c, W_{k+1}}) = \frac{5}{3}\mathcal{L}(\overline{W_{k+1}, A_2})$.

Then we find the coordinates of the control points:

$P_0 = W_0$,

$$P_a = \frac{16W_{k-1} - 5W_0 - 5W_k}{6} .$$

$P_b = W_k$,

$$P_c = \frac{16W_{k+1} - 5W_k - 5W_m}{6} .$$

$P_n = W_m$.

Such that W is interpolation point, P is control point and $\mathcal{L}(\overline{A, W})$ is a length of straight line between A and W.

Condition:

Measured angle $W_{k-1}W_0A_1$ is less than or equal to 90^{o}; so is measured angle $A_2W_mW_{k+1}$.

Remark:

1. The distance between interpolation point and the approximation
2. curve increases when interpolation point is increased.

4- Applications of Bézier Proposed Method in Image Processing

The image processing means treating the image of the occurred deformations, which may result from enlarging the image or in losing some its parts during the process of sending and receiving the image. In this section, we shall choose images randomly losing some of their parts, trying to construct this image by using interpolating Bézier curve**.**

(4-1) Proposed method for processing concealment block of image:

In this section, we shall acknowledge the relation of interpolation to find the concealment points, the way in which cure the concealment blocks. We shall identify the way to find one concealment pixel, by given sequence of data as 10,20,30,…etc, and represent the data of X due to the space between the pixels is equal. We get data (of row or column or main diameter or secondary diameter) in which the concealment pixel whereas represent the Y data , by finding the interpolation function for the resulted data. We could easily find the interpolation points with its image concealment pixel , so that it could get the approximate values for the concealment pixel [which give the range of these values to get the final value that represents the approximate pixel].

Before undertaking known cure concealment block, we shall define what concealment block is. The concealment block is group of concealment pixels, which usually take a form of square matrix. Currently, we shall clarify the proposed method of processing of concealment blocks of image by the following algorithm:

Step 1

1. We use the diameters which passed the corners of the concealment area for the image , find interpolation of each corner.
2. If has found other concealment pixels, then go to step 2 .
3. If There are no concealment pixels, then go to step 5 .

Step 2

1. Calculate the number of the concealment pixels in the columns and rows, and compare with the concealment pixels in the corners.
2. If the number of the pixels in the columns and rows more greater than that in corners, then go to step 1.
3. If its number in corners more bigger in rows and columns, then go to step3.

Step 3

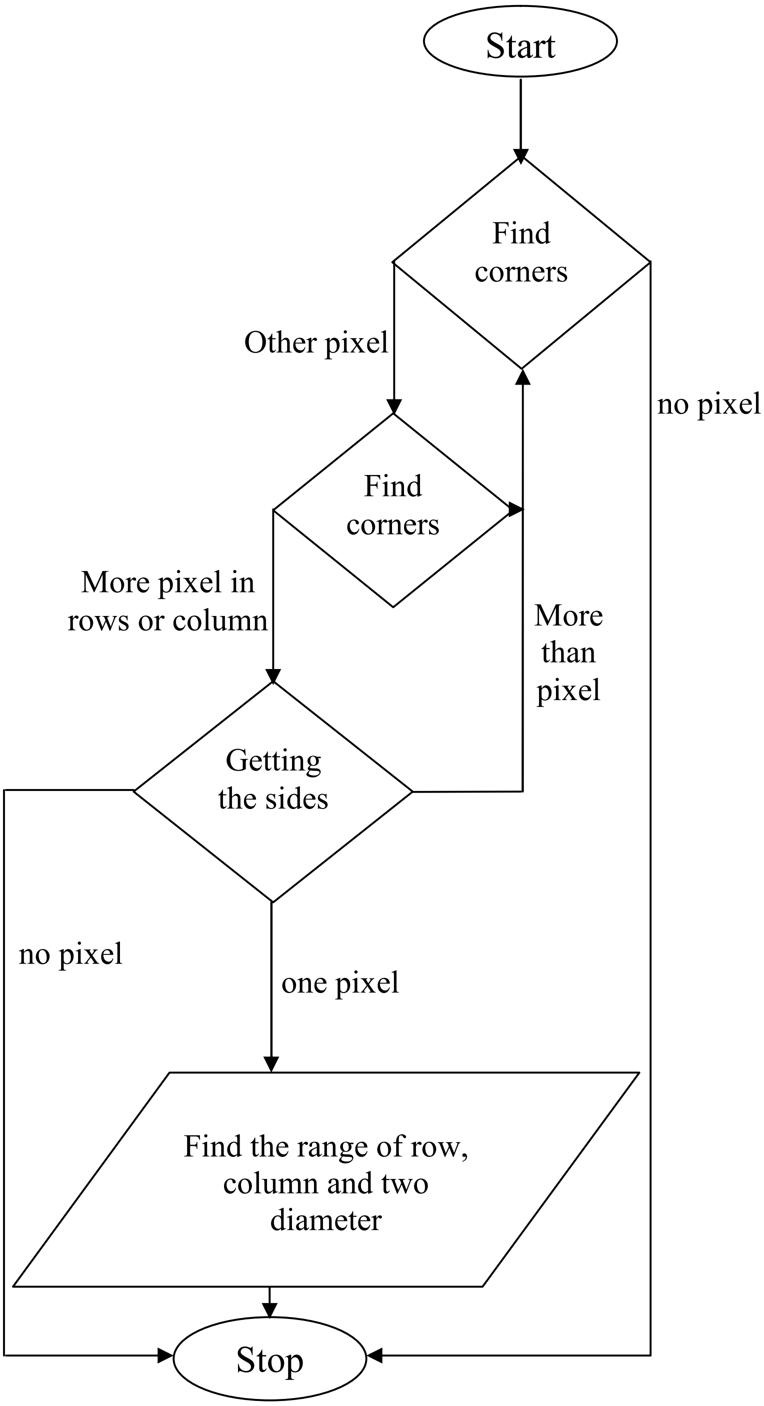
1. Getting the rows and columns for the concealment bocks, which pass through the concealment pixels and find each concealment area of the row and column.
2. If is available other concealment pixels (more than one pixel) , then go to step 1.
3. If There are no other available pixels, then go to step 5 .
4. If There is one pixel, then go to step 4 .

Step 4

1. Getting the row and column and the two diameter pasing through the pixel and find the interpolation for each one and find the range of the result which is the approximate value for the pixel .
2. Go to step 5 .

Step 5

Stop


Start
Find corners
Other pixel
no pixel
Find corners
More pixel in rows or column
More than pixel
Getting the sides
no pixel
one pixel
Find the range of row, column and two diameter
Stop

(4-2) Applied example:
we shall study the possibility of applying the proposed method for curing the concealment blocks on image whereas, we shall depend on the proposed function for interpolating Bézier to choose the process of interpolation using in this purpose.

Assumed that the neighboring child image is an applied example, whereas this image of type of grayscale with extension of BMP and size 128×128, wanted curing the concealment 8×8 block by applying the proposed methods.

Fig. 4.1 Child image

By using the algorithm mentioned in previous section, we shall find curing the concealment blocks, according to nine steps as following:

Step 1: we shall find four concealment points from the following interpolation points.

Above left corner .

x	**10**	**20**	**30**	**40**	**50**
y	153	150	?	147	140

By find interpolation function for above data via the proposed method, we get the first concealment point (150).

Above right corner .

x	**10**	**20**	**30**	**40**	**50**
y	127	127	?	139	144

By find interpolation function for above data via the proposed method, we get the first concealment point (132).

Under left corner .

x	**10**	**20**	**30**	**40**	**50**
y	156	156	?	164	172

By find interpolation function for above data via the proposed method, we get the first concealment point (160).

Under right corner .

x	10	20	30	40	50
y	116	128	?	151	156

By find interpolation function for above data via the proposed method, we get the first concealment point (141).

By the same of the way we can find step2 – step9, after entering the resulted values in matrix we can possible to observe the approximation resolution through Fig. 4.9 where part (a) represents the original image and (b) represents the approximate image.

(a) (b)

Fig. 4.2 The Original and Approximate Image

Conclusion:

We used Bézier curve in interpolation via finding the control points in a well arranged shape with interpolation points, where Bézier curve (defined by the control points) passed through the interpolation points. Through investigating the pervious studies in this field, it became clearer that the geometric method of finding the control points is more efficient than the analytic one, so we depended one of the geometric method to find the control points in the proposed method that the study deals with.

The proposed method adding some conditions, such that its depend on what is mentioned previously, and finding Bézier curve. Finally, we concluded a new algorithm by applying the proposed methods on treating the image concealment blocks to identify the way these interpolation methods work.

References

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