

## Connectedness in Čech Fuzzy Soft Closure Spaces

Lina H. Maibed

Rasha N. Majeed

Department of Mathematics , College of Education for pure sciences Abn Al-Haitham , University of Baghdad , Baghdad, Iraq  
Linah8324@gmail.com rashanm6@gmail.com

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### Abstract:

The notion of Čech fuzzy soft closure spaces was defined and its basic properties are introduced very newly by Majeed [1]. In the present paper, we define the notion of fuzzy soft separated sets in Čech fuzzy soft closure spaces and prove some properties concerning to this notion. By using the notion of fuzzy soft separated sets we introduce and study the concept of connected in both Čech fuzzy soft closure spaces and their associative fuzzy soft topological spaces. Then we introduce the concept of feebly connected, and discuss the relationship between the concepts of connected and feebly connected. Finally, we introduce several examples to clarify our results.

**Keywords.** Fuzzy soft set, Čech fuzzy soft closure operator, Fuzzy soft separated sets, Connected Čech fuzzy soft closure space, Feebly connected Čech fuzzy soft closure space.

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## 1. Introduction

It is known that Zadeh [2] in 1965 introduced the principal idea of fuzzy sets, which is supply a natural basis for handling mathematically the fuzzy phenomena which exist in our real world, and for constructing new branches of fuzzy mathematics. Later in 1999, Molodtsov [3] initiated the concept of soft set theory, which is a purely new way for modeling uncertainty. Molodtsov [3] established the main results of this new theory and successfully applied the soft set theory into several directions, such as theory of probability, Riemann integration, smoothness of functions, operations research and game theory. The concept of fuzzy soft sets was defined by Maji et al. [4] as fuzzy generalizations of soft sets. Then in 2011, Tanay and Kandemir [5] were gave the concept of topological structure based on fuzzy soft sets. The study of fuzzy soft topological spaces was pursued in recent years by some others [6, 7, 8, 9, 10, 11].

Čech [12] in 1966, introduced the notion of Čech closure spaces  $(X, \mathcal{C})$ , where  $\mathcal{C}: P(X) \rightarrow P(X)$  is a mapping satisfying  $\mathcal{C}(\emptyset) = \emptyset, A \subseteq \mathcal{C}(A)$  and  $\mathcal{C}(A \cup B) = \mathcal{C}(A) \cup \mathcal{C}(B)$ , the mapping  $\mathcal{C}$  called Čech closure operator on  $X$ . After Zadeh introduced the concept of fuzzy sets, in 1985 Mashhour and Ghanim [13] put the concept of Čech fuzzy closure spaces when they exchange sets by fuzzy sets in the definition of Čech closure space. In 2014, Gowri and Jegadeesan [14] using the concept of soft sets to introduced and investigation soft Čech closure spaces, the soft closure operator in that sense was defined from the power set  $P(X_{F_A})$  of  $X_{F_A}$  to itself (where  $F_A$  is a soft set over the universe set  $X$  with the set of parameter  $K$ , and  $A \subseteq K$ ). Also, in the same year, Krishnaveni and Sekar [15] introduced and study Čech soft closure spaces (where the soft closure operator here defined from the set of all soft sets over  $X$  to itself). Very recently Majeed [1] employ the fuzzy set theory to define and study the notion of Čech fuzzy soft closure spaces which is a generalization to Čech soft closure spaces that given by Krishnaveni and Sekar [15]. Also, Majeed and Maibed [16] introduced some structures of Čech fuzzy soft closure spaces. They show that every Čech fuzzy soft closure space gives a parameterized family of Čech fuzzy closure spaces, and defined and studied fuzzy soft exterior (respectively, boundary) in Čech fuzzy soft closure spaces.

On the other hand, the notion of connectedness in closure spaces is introduced and studied. Čech [12] defined the notion of connected spaces in closure spaces. According to Čech a subset  $A$  of a closure space  $X$  is said to be connected in  $X$  if  $A$  can not be represent as the union of two nonempty semi-separated subsets of  $X$ , that is  $A = A_1 \cup A_2$ ,  $(\mathcal{C}(A_1) \cap A_2) \cup (A_1 \cap \mathcal{C}(A_2)) = \emptyset$  implies  $A_1 = \emptyset$  or  $A_2 = \emptyset$ . Plastria [17] studied connectedness and local connectedness of simple extension. Gowri and Jegadeesan [18] introduced the concept of connectedness in soft Čech closure spaces.

In the present paper, we extend the notion of connectedness in Čech fuzzy soft closure spaces. In Section 3, we define the concept of fuzzy soft separated sets in Čech fuzzy soft closure spaces and give some of its basic properties. Then we introduce the notion of disconnected in both Čech fuzzy soft closure spaces and their associative fuzzy soft topological spaces based on fuzzy soft separated sets. In Section 4, we present the concept of feebly disconnected Čech fuzzy soft closure space. We show that the concept of disconnected and feebly disconnected are independent (see Examples 4.11 and 4.12).

## 2. Preliminaries

In this section we review some basic definitions and results related of fuzzy soft theory and Čech fuzzy soft closure spaces that will be needed in the sequel, and we foresee the reader be familiar with the usual notions and most basic ideas of fuzzy set theory. Throughout our paper,  $X$  will refer to the initial universe,  $I = [0,1]$ ,  $I_0 = (0,1]$ ,  $I^X$  be the set of all fuzzy sets of  $X$ , and  $K$  the set of parameters for  $X$ .

**Definition 2.1** [9, 10, 19, 20] A fuzzy soft set (fss, for short)  $\lambda_A$  on  $X$  is a mapping from  $K$  to  $I^X$ , i.e.,  $\lambda_A: K \rightarrow I^X$ , where  $\lambda_A(h) \neq \bar{0}$  if  $h \in A \subseteq K$  and  $\lambda_A(h) = \bar{0}$  if  $h \notin A \subseteq K$ , where  $\bar{0}$  is the empty fuzzy set on  $X$ . The family of all fuzzy soft sets over  $X$  denoted by  $\mathcal{F}_{ss}(X, K)$ .

In the next definition, the basic operations between fuzzy soft sets are given.

**Definition 2.2** [9, 10, 20] Let  $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$ , then

1.  $\lambda_A$  is said to be a fuzzy soft subset of  $\mu_B$ , denoted by  $\lambda_A \subseteq \mu_B$ , if  $\lambda_A(h) \leq \mu_B(h)$ , for all  $h \in K$ .
2.  $\lambda_A$  and  $\mu_B$  are said to be equal, denoted by  $\lambda_A = \mu_B$  if  $\lambda_A \subseteq \mu_B$  and  $\mu_B \subseteq \lambda_A$ .
3. The union of  $\lambda_A$  and  $\mu_B$ , denoted by  $\lambda_A \cup \mu_B$  is the fss  $\sigma_{(A \cup B)}$  defined by  $\sigma_{(A \cup B)}(h) = \lambda_A(h) \vee \mu_B(h)$ , for all  $h \in K$ .
4. The intersection of  $\lambda_A$  and  $\mu_B$ , denoted by  $\lambda_A \cap \mu_B$  is the fss  $\sigma_{(A \cap B)}$  defined by  $\sigma_{(A \cap B)}(h) = \lambda_A(h) \wedge \mu_B(h)$ , for all  $h \in K$ .

**Definition 2.3** [9, 11, 20] The null fss, denoted by  $\bar{0}_K$ , is a fss defined by  $\bar{0}_K(h) = \bar{0}$ , for all  $h \in K$ .

**Definition 2.4** [9, 11, 20] The universal fss, denoted by  $\bar{1}_K$ , is a fss defined by  $\bar{1}_K(h) = \bar{1}$ , for all  $h \in K$ , where  $\bar{1}$  is the universal fuzzy set of  $X$ .

**Definition 2.5** [20] The complement of a fss  $\lambda_A \in \mathcal{F}_{ss}(X, K)$ , denoted  $\bar{1}_K - \lambda_A$ , is the fss defined by  $(\bar{1}_K - \lambda_A)(h) = \bar{1} - \lambda_A(h)$ , for each  $h \in K$ , Its clear that  $\bar{1}_K - (\bar{1}_K - \lambda_A) = \lambda_A$ .

**Definition 2.6** [21] Two fss's  $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$  are said to be disjoint, denoted by  $\lambda_A \cap \mu_B = \bar{0}_K$ , if  $\lambda_A(h) \cap \mu_B(h) = \bar{0}$  for all  $h \in K$ .

**Definition 2.7** [5, 20] A fuzzy soft topological space (fst, for short)  $(X, \tau, K)$  where  $X$  is a nonempty set with a fixed set of parameters and  $\tau$  is a family of fuzzy soft sets over  $X$  satisfying the following properties:

1.  $\bar{0}_K, \bar{1}_K \in \tau$ ,
2. If  $\lambda_A, \mu_B \in \tau$ , then  $\lambda_A \cap \mu_B \in \tau$ ,
3. If  $(\lambda_A)_i \in \tau$ , then  $\cup_{i \in J} (\lambda_A)_i \in \tau$ .

$\tau$  is called a topology of fuzzy soft sets on  $X$ . Every member of  $\tau$  is called open fuzzy soft set (open-fss, for short). The complement of open-fss is called a closed fuzzy soft set (closed-fss, for short).

**Definition 2.8** [1] An operator  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  is called Čech fuzzy soft closure operator (Č-fsco, for short) on  $X$ , if the following axioms are satisfied.

- (C1)  $\theta(\bar{0}_K) = \bar{0}_K$ ,
- (C2)  $\lambda_A \subseteq \theta(\lambda_A)$ , for all  $\lambda_A \in \mathcal{F}_{ss}(X, K)$ ,
- (C3)  $\theta(\lambda_A \cup \mu_B) = \theta(\lambda_A) \cup \theta(\mu_B)$ , for all  $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$ .

The triple  $(X, \theta, K)$  is called a Čech fuzzy soft closure space (ČF-fsccs, for short).

A fss  $\lambda_A$  is said to be closed-fss in  $(X, \theta, K)$  if  $\lambda_A = \theta(\lambda_A)$ . And a fss  $\lambda_A$  is said to be an open-fss if  $\bar{1}_K - \lambda_A$  is a closed-fss.

**Proposition 2.9** [1] Let  $(X, \theta, K)$  be a ČF-scs, and  $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$  such that  $\lambda_A \subseteq \mu_B$ , then  $\theta(\lambda_A) \subseteq \theta(\mu_B)$ .

**Definition 2.10** [1] Let  $(X, \theta, K)$  be a ČF-scs, and let  $\lambda_A \in \mathcal{F}_{ss}(X, K)$ . The interior of  $\lambda_A$ , denoted by  $Int(\lambda_A)$  is defined as  $Int(\lambda_A) = \bar{1}_K - (\theta(\bar{1}_K - \lambda_A))$ .

**Definition 2.11** [1] Let  $V$  be a non-empty subset of  $X$ , then  $\bar{V}_K$  denotes the fuzzy soft set  $V_K$  over  $X$  for which  $V(h) = \bar{1}_V$  for all  $h \in K$ , (where  $\bar{1}_V: X \rightarrow I$  such that  $\bar{1}_V(x) = 1$  if  $x \in V$  and  $\bar{1}_V(x) = 0$  if  $x \notin V$ ).

**Theorem 2.12** [1] Let  $(X, \theta, K)$  be a ČF-scs,  $V \subseteq X$  and let  $\theta_V: \mathcal{F}_{ss}(V, K) \rightarrow \mathcal{F}_{ss}(V, K)$  defined as  $\theta_V(\lambda_A) = \bar{V}_K \cap \theta(\lambda_A)$ . Then  $\theta_V$  is a ČF-sco. The triple  $(V, \theta_V, K)$  is said to be Čech fuzzy soft closure subspace (ČF-sc subspace, for short) of  $(X, \theta, K)$ .

**Theorem 2.13** [1] Let  $(X, \theta, K)$  be a ČF-scs and let  $\tau_\theta \subseteq \mathcal{F}_{ss}(X, K)$ , defined as follows

$$\tau_\theta = \{\bar{1}_K - \lambda_A : \theta(\lambda_A) = \lambda_A\}.$$

Then  $\tau_\theta$  is a fuzzy soft topology on  $X$  and  $(X, \tau_\theta, K)$  is called an associative fst of  $(X, \theta, K)$ .

**Definition 2.14** [22] Let  $(X, \tau_\theta, K)$  be an associative fst of  $(X, \theta, K)$  and let  $\lambda_A \in \mathcal{F}_{ss}(X, K)$ . The fuzzy soft topological clouser of  $\lambda_A$  with respect to  $\theta$ , denoted by  $\tau_\theta-cl(\lambda_A)$ , is the intersection of all closed fuzzy soft super sets of  $\lambda_A$ . i.e.,

$$\tau_\theta-cl(\lambda_A) = \cap \{\rho_C : \lambda_A \subseteq \rho_C \text{ and } \theta(\rho_C) = \rho_C\}. \quad (2.1)$$

And, The fuzzy soft topological interior of  $\lambda_A$  with respect to  $\theta$ , denoted by  $\tau_\theta-int(\lambda_A)$  is the union of all open fuzzy soft subset of  $\lambda_A$ . i.e.,

$$\tau_\theta-int(\lambda_A) = \cup \{\rho_C : \rho_C \subseteq \lambda_A \text{ and } \theta(\bar{1}_K - \rho_C) = \bar{1}_K - \rho_C\}. \quad (2.2)$$

The next theorem give the relation between the Č-fsco  $\theta$  (respectively, interior operator  $Int$ ) and the fuzzy soft topological closure  $\tau_\theta-cl$  (respectively, interior  $\tau_\theta-int$ ).

**Theorem 2.15** [22] Let  $(X, \theta, K)$  be ČF-scs and  $(X, \tau_\theta, K)$  be an associative fst of  $(X, \theta, K)$ . Then for any  $\lambda_A \in \mathcal{F}_{ss}(X, K)$

$$\tau_\theta-int(\lambda_A) \subseteq Int(\lambda_A) \subseteq \lambda_A \subseteq \theta(\lambda_A) \subseteq \tau_\theta-cl(\lambda_A). \quad (2.3)$$

### 3.Connected Čech Fuzzy Soft Closure Spaces

In this section we introduce and study fuzzy soft separated sets in ČF-scs, then we use it to introduce the notion of connectedness in ČF-scs's.

**Definition 3.1** Let  $(X, \theta, K)$  be a ČF-scs. If there exist non-empty proper fss's  $\lambda_A, \mu_B \in \mathcal{F}_{ss}(X, K)$ , such that  $\lambda_A \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\lambda_A) \cap \mu_B = \bar{0}_K$ , then the fss's  $\lambda_A$  and  $\mu_B$  are called fuzzy soft separated sets.

In other words, two non-empty fuzzy soft set  $\lambda_A, \mu_B$  of ČF-scs  $(X, \theta, K)$  are said to be fuzzy soft separated sets if and only if  $(\lambda_A \cap \theta(\mu_B)) \cup (\theta(\lambda_A) \cap \mu_B) = \bar{0}_K$ .

**Remark 3.2** It is clear that if  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \theta, K)$ , then  $\lambda_A$  and  $\mu_B$  are disjoint fuzzy soft sets. The following example shows that the converse is not true.

**Example 3.3** Let  $X=\{a, b, c\}$ ,  $K=\{h_1, h_2\}$  and let  $\rho_C = \{(h_1, b_{0.5}), (h_2, b_{0.5})\}$ . Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_{0.5} \vee b_{0.5}), (h_{0.5}, a_{0.5} \vee b_{0.5})\} & \text{if } \lambda_A \subseteq \rho_C, \\ \bar{1}_K & \text{otherwise.} \end{cases}$$

Then  $\theta$  is Č-fsco on  $X$ . Here we have  $\lambda_A = \{(h_1, b_{0.5})\}$  and  $\mu_B = \{(h_1, a_{0.5}), (h_2, c_{0.5})\}$  are non-empty disjoint fuzzy soft sets but  $\lambda_A$  and  $\mu_B$  are not fuzzy soft separated sets.

**Theorem 3.4** Let  $(X, \theta, K)$  be a ČF-scs. Then every fuzzy soft subset of fuzzy soft separated sets are also fuzzy soft separated sets.

**Proof.** Let  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \theta, K)$ , and let  $\rho_C \subseteq \lambda_A$  and  $\eta_D \subseteq \mu_B$ . Since  $\rho_C \subseteq \lambda_A$  and  $\eta_D \subseteq \mu_B$ , then by Proposition 2.9, we have  $\theta(\rho_C) \subseteq \theta(\lambda_A)$  and  $\theta(\eta_D) \subseteq \theta(\mu_B)$ . This implies  $\theta(\rho_C) \cap \theta(\eta_D) \subseteq \theta(\lambda_A) \cap \theta(\mu_B)$  and  $\theta(\eta_D) \cap \rho_C \subseteq \theta(\mu_B) \cap \lambda_A$ . But  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets, it follows  $\theta(\rho_C) \cap \theta(\eta_D) \subseteq \theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\eta_D) \cap \rho_C \subseteq \theta(\mu_B) \cap \lambda_A = \bar{0}_K$ . Hence  $\theta(\rho_C) \cap \theta(\eta_D) = \bar{0}_K$  and  $\theta(\eta_D) \cap \rho_C = \bar{0}_K$ . Thus  $\rho_C$  and  $\eta_D$  are fuzzy soft separated sets. ■

**Theorem 3.5** Let  $(V, \theta_V, K)$  be a ČF-sc subspace of  $(X, \theta, K)$  and let  $\lambda_A, \mu_B \in \mathcal{F}_{ss}(V, K)$ , then  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \theta, K)$  if and only if  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(V, \theta_V, K)$ .

**Proof.** Let  $(X, \theta, K)$  be a ČF-scs and  $(V, \theta_V, K)$  be a ČF-sc subspace of  $(X, \theta, K)$ . Assume that that  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \theta, K)$ , this implies that  $\lambda_A \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\lambda_A) \cap \mu_B = \bar{0}_K$ . Which means  $(\lambda_A \cap \theta(\mu_B)) \cup (\theta(\lambda_A) \cap \mu_B) = \bar{0}_K$ .

$$\begin{aligned} \text{Now,} \\ (\lambda_A \cap \theta_V(\mu_B)) \cup (\theta_V(\lambda_A) \cap \mu_B) &= (\lambda_A \cap (\bar{V}_K \cap \theta(\mu_B))) \\ \cup ((\bar{V}_K \cap \theta(\lambda_A)) \cap \mu_B) \\ &= ((\lambda_A \cap \bar{V}_K) \cap \theta(\mu_B)) \cup ((\bar{V}_K \cap \mu_B) \cap \theta(\lambda_A)) \\ &= (\lambda_A \cap \theta(\mu_B)) \\ \cup (\mu_B \cap \theta(\lambda_A)) &= \bar{0}_K. \end{aligned}$$

Therefore,  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \theta, K)$  if and only if  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(V, \theta_V, K)$ . ■

**Definition 3.6** A ČF-scs  $(X, \theta, K)$  is said to be disconnected Čech fuzzy soft closure space (disconnected-ČF-scs, for short) if there exist fuzzy soft separated sets  $\lambda_A$  and  $\mu_B$  such that  $\theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$ .

**Definition 3.7** A ČF-scs  $(X, \theta, K)$  is said to be connected Čech fuzzy soft closure space (connected-ČF-scs, for short) if it is not disconnected-ČF-scs.

Now we give two examples one is disconnected-ČF-scs and the other is connected-ČF-scs.

**Example 3.8** Let  $X=\{a, b\}$ ,  $K=\{h_1, h_2\}$ . Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1)\} & \text{if } \lambda_A \subseteq \{(h_1, a_1)\}, \\ \{(h_2, a_1 \vee b_1)\} & \text{if } \lambda_A \subseteq \{(h_2, a_1)\}, \\ \bar{1}_K & \text{other wise.} \end{cases}$$

Then  $(X, \theta, K)$  is disconnected-ČF-scs. To explain that taking  $\lambda_A = \{(h_1, a_{0.5})\}$  and  $\mu_B = \{(h_2, a_{0.2})\}$ . It is clear that  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets such that  $\theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$ .

**Example 3.9** Let  $X=\{a, b\}$ ,  $K=\{h_1, h_2\}$ . Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1)\} & \text{if } \lambda_A \subseteq \{(h_1, a_1 \vee b_1)\}, \\ \bar{1}_K & \text{otherwise.} \end{cases}$$

Then  $(X, \theta, K)$  is connected- $\check{\mathcal{F}}$ -scs.

**Remark 3.10** Connectedness in  $\check{\mathcal{F}}$ -scs is not hereditary property. The following example explain that.

**Example 3.11** Let  $X=\{a, b, c\}$ ,  $K=\{h_1, h_2\}$  and let  $(\lambda_A)_1, (\lambda_A)_2 \in \mathcal{F}_{ss}(X, K)$  such that

$$(\lambda_A)_1 = \{(h_1, a_1 \vee b_1 \vee c_{0.4})\} \text{ and } (\lambda_A)_2 = \{(h_2, a_1 \vee b_1 \vee c_{0.7})\}.$$

Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1 \vee c_{0.4})\} & \text{if } \lambda_A \subseteq (\lambda_A)_1, \\ \{(h_2, a_1 \vee b_1 \vee c_{0.7})\} & \text{if } \lambda_A \subseteq (\lambda_A)_2, \\ \theta((\lambda_A)_1) \cup \theta((\lambda_A)_2) & \text{if } \lambda_A \subseteq (\lambda_A)_1 \cup (\lambda_A)_2, \\ \bar{1}_K & \text{otherwise.} \end{cases}$$

Then  $(X, \theta, K)$  is connected- $\check{\mathcal{F}}$ -scs. Let  $V = \{a, b\}$ , then  $\theta_V: \mathcal{F}_{ss}(V, K) \rightarrow \mathcal{F}_{ss}(V, K)$  defined as

$$\theta_V(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1)\} & \text{if } \lambda_A \subseteq \{(h_1, a_1 \vee b_1)\}, \\ \{(h_2, a_1 \vee b_1)\} & \text{if } \lambda_A \subseteq \{(h_2, a_1 \vee b_1)\}, \\ \bar{V}_K & \text{otherwise.} \end{cases}$$

Then  $(V, \theta_V, K)$  is disconnected- $\check{\mathcal{F}}$ -sc subspace of  $(X, \theta, K)$ . Since there exist  $\lambda_A = \{(h_1, a_1 \vee b_1)\}$  and  $\mu_B = \{(h_2, a_1 \vee b_1)\}$  are fuzzy soft separated sets such that  $\theta_V(\lambda_A) \cap \theta_V(\mu_B) = \bar{0}_K$  and  $\theta_V(\lambda_A) \cup \theta_V(\mu_B) = \bar{V}_K$ .

Now, we introduce the concept of fuzzy soft separated sets in the associative fsts's of  $\check{\mathcal{F}}$ -scs's.

**Definition 3.12** Two non-empty fss's  $\lambda_A$  and  $\mu_B$  are said to be fuzzy soft separated sets in the associative fsts  $(X, \tau_\theta, K)$ , if  $\lambda_A \cap \tau_\theta - c(\mu_B) = \bar{0}_K$  and  $\tau_\theta - c(\lambda_A) \cap \mu_B = \bar{0}_K$ .

**Theorem 3.13** If  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in the associative fsts  $(X, \tau_\theta, K)$ , then  $\lambda_A$  and  $\mu_B$  are also fuzzy soft separated sets in  $(X, \theta, K)$ .

**Proof.** Let  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \tau_\theta, K)$ . Then  $\lambda_A \cap \tau_\theta - c(\mu_B) = \bar{0}_K$  and  $\tau_\theta - c(\lambda_A) \cap \mu_B = \bar{0}_K$ . By Theorem 2.15, we get,  $\lambda_A \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\lambda_A) \cap \mu_B = \bar{0}_K$ . This implies  $\lambda_A$  and  $\mu_B$  are fuzzy soft separated sets in  $(X, \theta, K)$ . ■

**Definition 3.14** An associative fsts  $(X, \tau_\theta, K)$  of  $\check{\mathcal{F}}$ -scs  $(X, \theta, K)$  is said to be disconnected fsts, if there exist two fuzzy soft separated sets  $\lambda_A$  and  $\mu_B$  in  $(X, \tau_\theta, K)$  such that  $\tau_\theta - c(\lambda_A) \cap \tau_\theta - c(\mu_B) = \bar{0}_K$  and  $\tau_\theta - c(\lambda_A) \cup \tau_\theta - c(\mu_B) = \bar{1}_K$ .

**Definition 3.15** An associative fsts  $(X, \tau_\theta, K)$  of  $\check{\mathcal{F}}$ -scs  $(X, \theta, K)$  is said to be connected fsts, if it is not disconnected fsts.

**Theorem 3.16** If  $(X, \tau_\theta, K)$  is a disconnected fsts, then  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs.

**Proof.** Let  $(X, \tau_\theta, K)$  be disconnected fsts, then there exist two fuzzy soft separated sets  $\lambda_A$  and  $\mu_B$  in  $(X, \tau_\theta, K)$  such that  $\tau_\theta - c(\lambda_A) \cap \tau_\theta - c(\mu_B) = \bar{0}_K$  and  $\tau_\theta - c(\lambda_A) \cup \tau_\theta - c(\mu_B) = \bar{1}_K$ . Since  $\tau_\theta - c(\lambda_A)$  and  $\tau_\theta - c(\mu_B)$  are closed-fss's, then  $\theta(\tau_\theta - c(\lambda_A)) = \tau_\theta - c(\lambda_A)$  and  $\theta(\tau_\theta - c(\mu_B)) = \tau_\theta - c(\mu_B)$ . Let  $\rho_C = \tau_\theta - c(\lambda_A)$  and  $\eta_D = \tau_\theta - c(\mu_B)$ . Then we have  $\rho_C$  and  $\eta_D$  are fuzzy soft separated sets in  $(X, \theta, K)$  such that  $\theta(\rho_C) \cap \theta(\eta_D) = \rho_C \cap \eta_D = \bar{0}_K$  and  $\theta(\rho_C) \cup \theta(\eta_D) = \rho_C \cup \eta_D = \bar{1}_K$ . Hence,  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs. ■

**Corollary 3.17** If  $(X, \theta, K)$  is connected- $\check{\mathcal{F}}$ -scs, then  $(X, \tau_\theta, K)$  is a connected fsts.

**Proof.** The proof follows by suppose  $(X, \tau_\theta, K)$  is disconnected fsts. From Theorem 3.16, we get  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs which is a contradiction with hypothesis. Hence, the result. ■

**Remark 3.18** The converse of Theorem 3.16 and its corollary is not true in general. That is, if  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs, then  $(X, \tau_\theta, K)$  need not to disconnected fsts. The following example shows that.

**Example 3.19** In Example 3.8,  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs. But its associative fsts  $(X, \tau_\theta, K)$  is connected fsts, because  $\tau_\theta = \{\bar{0}_K, \bar{1}_K\}$ .



#### 4. Feebly Connected Čech Fuzzy Soft Closure Spaces

**Definition 4.1** A  $\check{\mathcal{F}}$ -scs  $(X, \theta, K)$  is said to be feebly disconnected- $\check{\mathcal{F}}$ -scs, if there two non-empty disjoint fuzzy soft sets  $\lambda_A$  and  $\mu_B$  such that  $\lambda_A \cup \theta(\mu_B) = \bar{1}_K$  and  $\theta(\lambda_A) \cup \mu_B = \bar{1}_K$ .

**Definition 4.2** A  $\check{\mathcal{F}}$ -scs  $(X, \theta, K)$  is said to be feebly connected- $\check{\mathcal{F}}$ -scs if it is not feebly disconnected- $\check{\mathcal{F}}$ -scs.

**Remark 4.3** Feebly disconnectedness in  $\check{\mathcal{F}}$ -scs is not hereditary property. The following example explains that.

**Example 4.4** Let  $X = \{a, b, c\}$ ,  $K = \{h_1, h_2\}$  and let  $(\lambda_A)_1, (\lambda_A)_2 \in \mathcal{F}_{ss}(X, K)$  such that

$$(\lambda_A)_1 = \{(h_1, a_1 \vee c_1)\} \text{ and } (\lambda_A)_2 = \{(h_1, b_1), (h_2, a_1 \vee b_1 \vee c_1)\}.$$

Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee c_1)\} & \text{if } \lambda_A \subseteq (\lambda_A)_1, \\ \{(h_1, b_1), (h_2, a_1 \vee b_1 \vee c_1)\} & \text{if } \lambda_A \subseteq (\lambda_A)_2, \\ \bar{1}_K & \text{otherwise.} \end{cases}$$

Then  $(X, \theta, K)$  is feebly disconnected- $\check{\mathcal{F}}$ -scs. Since there exist  $\lambda_A = \{(h_1, a_1 \vee c_1)\}$  and  $\mu_B = \{(h_1, b_1), (h_2, a_1 \vee b_1 \vee c_1)\}$  are disjoint fuzzy soft sets such that  $\theta(\mu_B) \cup \lambda_A = \bar{1}_K$  and  $\mu_B \cup \theta(\lambda_A) = \bar{1}_K$ .

Let  $V = \{b\}$ , then  $\theta_V: \mathcal{F}_{ss}(V, K) \rightarrow \mathcal{F}_{ss}(V, K)$  defined as:

$$\theta_V(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \bar{V}_K & \text{otherwise.} \end{cases}$$

Then  $(V, \theta_V, K)$  is feebly connected- $\check{\mathcal{F}}$ -sc subspace of  $(X, \theta, K)$ .

**Definition 4.5** An associative fsts  $(X, \tau_\theta, K)$  of  $\check{\mathcal{F}}$ -scs  $(X, \theta, K)$  is said to be feebly disconnected fsts, if there exist two non-empty disjoint fuzzy soft sets  $\lambda_A$  and  $\mu_B$  such that  $\lambda_A \cup \tau_\theta\text{-}c(\mu_B) = \bar{1}_K$  and  $\tau_\theta\text{-}c(\lambda_A) \cup \mu_B = \bar{1}_K$ .

**Theorem 4.6** If  $(X, \theta, K)$  is feebly disconnected- $\check{\mathcal{F}}$ -scs, then  $(X, \tau_\theta, K)$  is feebly disconnected fsts.

**Proof.** The proof follows from the definition 4.1 and Theorem 2.19. ■

**Corollary 4.7** If  $(X, \tau_\theta, K)$  is feebly connected fsts, then  $(X, \theta, K)$  is feebly connected- $\check{\mathcal{F}}$ -scs.

**Proof.** The proof follows by suppose  $(X, \theta, K)$  is feebly disconnected- $\check{\mathcal{F}}$ -scs. From Theorem 4.6, we get  $(X, \tau_\theta, K)$  is feebly disconnected fsts which is a contradiction with hypothesis. Hence, the result. ■

Next we discuss the relationship between disconnectedness and feebly disconnectedness in  $\check{\mathcal{F}}$ -scs's.

**Remark 4.10** The concept of disconnected- $\check{\mathcal{F}}$ -scs and feebly disconnected- $\check{\mathcal{F}}$ -scs are independent. The next two examples explain our clime.

The following example shows that if  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs, then  $(X, \theta, K)$  need not to be feebly disconnected- $\check{\mathcal{F}}$ -scs.

**Example 4.11** Let  $X = \{a, b\}$ ,  $K = \{h\}$ . Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h, a_1)\} & \text{if } \lambda_A = \{(h, a_t); 0 < t < 1\}, \\ \{(h, b_1)\} & \text{if } \lambda_A = \{(h, b_s); 0 < s < 1\}, \\ \bar{1}_K & \text{other wise.} \end{cases}$$

Then  $(X, \theta, K)$  is disconnected- $\check{\mathcal{F}}$ -scs, since there exist  $\lambda_A = \{(h, a_{0.5})\}$  and  $\mu_B = \{(h, b_{0.3})\}$  are fuzzy soft separated sets such that  $\theta(\lambda_A) \cap \theta(\mu_B) = \bar{0}_K$  and  $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$ . However,  $(X, \theta, K)$  is not feebly disconnected- $\check{\mathcal{F}}$ -scs since for any non-empty disjoint fss's  $\lambda_A$  and  $\mu_B$ , we have  $\lambda_A \cup \theta(\mu_B) \neq \bar{1}_K$ .

The next example shows that if  $(X, \theta, K)$  is feebly disconnected- $\check{\mathcal{F}}$ -scs, then  $(X, \theta, K)$  need not to be disconnected- $\check{\mathcal{F}}$ -scs.

**Example 4.12** Let  $X = \{a, b\}$ ,  $K = \{h_1, h_2\}$ . Define  $\theta: \mathcal{F}_{ss}(X, K) \rightarrow \mathcal{F}_{ss}(X, K)$  as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K, \\ \{(h_1, a_1 \vee b_1), (h_2, b_1)\} & \text{if } \lambda_A \subseteq \{(h_1, b_1)\}, \\ \{(h_1, a_1), (h_2, a_1 \vee b_1)\} & \text{if } \lambda_A \subseteq \{(h_2, a_1)\}, \\ \bar{1}_K & \text{other wise.} \end{cases}$$

Then  $(X, \theta, K)$  is feebly disconnected- $\check{\mathcal{F}}$ -scs. Since there are non-empty disjoint fuzzy soft sets  $\lambda_A = \{(h_1, b_1)\}$  and  $\mu_B = \{(h_2, a_1)\}$  such that  $\theta(\lambda_A) \cup \mu_B = \bar{1}_K$  and  $\lambda_A \cup \theta(\mu_B) = \bar{1}_K$ .

And  $(X, \theta, K)$  is connected- $\check{\mathcal{F}}$ -scs. Since for any fuzzy soft separated sets  $\lambda_A$  and  $\mu_B$ , we have  $\theta(\lambda_A) \cup \theta(\mu_B) = \bar{1}_K$  but  $\theta(\lambda_A) \cap \theta(\mu_B) \neq \bar{0}_K$ .

**Remark 4.13** It is worth noting that the definitions of disconnected- $\check{C}\mathcal{F}$ -scs and feebly disconnected- $\check{C}\mathcal{F}$ -scs (see Definitions 3.6 and 4.1, respectively) turn to be every disconnected- $\check{C}\mathcal{F}$ -scs is feebly disconnected- $\check{C}\mathcal{F}$ -scs, if the fuzzy soft separated sets which are satisfying the conditions of disconnected- $\check{C}\mathcal{F}$ -scs are closed-fss's.

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## الاتصال في فضاءات الاغلاق الضبابية الناعمة من النوع – تشيك

لينا حسين معيب رشاش ناصر مجيد

قسم الرياضيات، كلية التربية للعلوم الصرفة ابن الهيثم، جامعة بغداد، بغداد، العراق.

### المستخلص :

يعتبر مفهوم فضاءات الاغلاق الضبابية الناعمة من النوع –تشيك من المفاهيم الحديثة حيث تم تعريفه ودراسة خواصه من قبل مجيد [1] . في هذا البحث قمنا بتعريف ودراسة مفهوم المجموعات الضبابية الناعمة القابلة للفصل في فضاءات الاغلاق الضبابية الناعمة من النوع – تشيك. باستخدام المجموعات الضبابية القابلة للفصل تم تعريف ودراسة مفهوم الاتصال في كلا من فضاءات الاغلاق الضبابية الناعمة من النوع –تشيك والفضاء الضبابي الناعم المشتق منه. كذلك عرفنا مفهوم الاتصال الضعيف ودرسنا العلاقة بين مفهوم الاتصال ومفهوم الاتصال الضعيف . واخيرا، اعطينا العديد من الامثلة لتوضيح النتائج التي تم التوصل اليها في البحث.

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