MODIFIED MID-POINTMETHOD METHOD FOR SOLVING LINEAR FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

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Abstract:

In this paper we use the Modified mid-pointmethod for solving the linear Fredholm integral equations of the second kind. In numerical examples we showed the effectiveness of this method, in comparison with other numerical methods such as, midpoint, trapezoidal, Simpson's and Modified trapezoidal methods.

1. Introduction:

The theory and application of integral equations is an important subject within applied mathematics. Integral equations are used as mathematical models for many and varied physical situations, and also occur as reformulations of other mathematical problems. Many physical problems are modeled by integral equations the numerical solutions of such equations have been highly studied by many authors, [1-4]. In recent years, numerous works have been focusing on the development of more advanced and efficient methods for integral equations such as Taylor-series method [5], Neumann series method [6], Decompostion method [7], Homotopy perturbation method [7, 8] and Homotopy analysis method [9, 10].

In recent years a number of authors have considered Modified quadrature rules, and applications it in numerical integration. For example, the Modified trapezoidal, Modified mid-point and Modified Simpson quadrature rules are considered in [11-15, 19, 21]. Sometimes they have considered generalizations of these rules in [16, 17, 20]. Some authors used Modified quadrature rules in numerical solutions of integral equations, such as Nadjafi in [22] used Modified trapezoidal method for solving integral equations, Majeed in [23] used this method for solving system of integral equations.

In this paper we use the composted Modified mid-pointrule

$$\int_a^b f(x)dx = \left(h\sum_{j=1}^{n} f(x_j) + \frac{h^2}{24}\left[f'(b) - f'(a)\right] \right) + \frac{7}{5760}(b-a)h^4 f^{(4)}(\xi). \qquad ..(1.1)$$

where n is the number of subinterval of [a, b] , $h = \frac{b-a}{n}$, $x_j = a + (j-1/2)h, j = 1,2,...,n$, to solve linear Fredholm integral equations of the second kind given by

$$u(x) = f(x) + \lambda\int_a^b k(x,y)u(y)dy, \;\; a \le x \le b \qquad \ldots\ldots\ldots\ldots\ldots (1.2)$$

where λ is a scalar parameter, f(x) and k(x,y) are given continuous functions, whereas u(x) is to be determined.

2- The Modified mid-pointmethod:

We consider the linear Fredholm integral equation of the second kind :

$$u(x) = f(x) + \int_a^b k(x,y)u(y)dy, \ a \le x \le b \qquad \text{……...…..(2.1)}$$

where, k(x,y) and f(x) is a differentiable functions with respect to their variables, i.e. the functions $\frac{\partial k(x,y)}{\partial x}$, $\frac{\partial k(x,y)}{\partial y}$ and $f'(x)$ exist.

To solve this equation, we approximate the integral part that appeared in the right hand side by the composed Modified mid-pointrule to get,

$$u(x) = f(x) + h\sum_{j=1}^{n} k(x,x_j)u(x_j) + \frac{h^2}{24}\big(J(x,x_{n+1/2})u(x_{n+1/2}) + k(x,x_{n+1/2})u'(x_{n+1/2}) -$$

$$J(x,x_{1/2})u(x_{1/2}) - k(x,x_{1/2})u'(x_{1/2})\big), \qquad \text{.....……… (2.2)}$$

where, $x_{1/2} = a$, $x_{n+1/2} = b$ and $J(x,y) = \frac{\partial k(x,y)}{\partial y}$.

Hence for $x = x_{1/2}, x_1, \ldots, x_n, x_{n+1/2}$ we get the following system of equations:

$$u_i = f_i + h\sum_{j=1}^{n} k_{i,j}u_j + \frac{h^2}{24}\big(J_{i,n+1/2}u_{n+1/2} + k_{i,n+1/2}u'_{n+1/2} - J_{i,1/2}u_{1/2} - k_{i,1/2}u'_{1/2}\big)\Big],$$

$$\text{……..…….. (2.3)}$$

where $u_i = u(x_i)$, $u'_i = u'(x_i)$, $f_i = f(x_i)$, $k_{i,j} = k(x_i, x_j)$ and $J_{i,j} = J(x_i, x_j)$.

If we differentiate both sides of equation (2.1) with respect to x and setting $H(x,y) = \frac{\partial k(x,y)}{\partial x}$ one can obtain:

$$u'(x) = f'(x) + \int_a^b H(x,y)u(y)dy, \ a \le x \le b, \qquad \text{...………..... (2.4)}$$

We note that if u is a solution of eq.(2.1) then it is a solution of eq.(2.4) too.

Now, for solving eq.(2.4), we must consider two cases.

Case 1: The partial derivatives $L(x,y) = \frac{\partial^2 k(x,y)}{\partial x \partial y}$ exist. In this case, we approximate the integral part that appeared in the right hand side of eq.(2.4) by the composed modified mid-pointrule to get,

$$u'(x) = f'(x) + h\sum_{j=1}^{n} H(x,x_j)u(x_j) + \frac{h^2}{24}\big(L(x,x_{n+1/2})u(x_{n+1/2}) + H(x,x_{n+1/2})u'(x_{n+1/2}) -$$

$$L(x,x_{1/2})u(x_{1/2}) - H(x,x_{1/2})u'(x_{1/2})\big)\Big], \qquad \text{..…………. (2.5)}$$

By setting $x = x_{1/2}, x_{n+1/2}$, in eq.(2.5), one can get

$$u'_i = f'_i + h\sum_{j=1}^{n} H_{i,j}u_j + \frac{h^2}{24}\big(L_{i,n+1/2}u_{n+1/2} + H_{i,n+1/2}u'_{n+1/2} - L_{i,1/2}u_{1/2} - H_{i,1/2}u'_{1/2}\big)\Big],$$

$$i = 1/2, n+1/2. \qquad \text{………..…. (2.6)}$$

where $f'_i = f'(x_i)$, $H_{i,j} = H(x_i, x_j)$ and $L_{i,j} = L(x_i, x_j)$.

From eq.(2.6) and eq.(2.3) we get the following linear system

$$AU = F \qquad \text{………..…. (2.7)}$$

of (n+4) equations in (n+4) unknowns $\{u_i\}, \{u'_{1/2}\}$ and $\{u'_{n+1/2}\}$, $i = 1/2, 1, 2, \ldots, n, n+1/2$, where A, U, F are matrices defined by

$$A=\left[\begin{array}{cccccc|cc}
1+\frac{h^2}{24}J_{\frac{1}{2},\frac{1}{2}} & -hk_{\frac{1}{2},1} & -hk_{\frac{1}{2},2} & \cdots & -hk_{\frac{1}{2},n} & -\frac{h^2}{24}J_{\frac{1}{2},n+\frac{1}{2}} & \frac{h^2}{24}k_{\frac{1}{2},\frac{1}{2}} & -\frac{h^2}{24}k_{\frac{1}{2},n+\frac{1}{2}} \\
\frac{h^2}{24}J_{1,\frac{1}{2}} & 1-hk_{1,1} & -hk_{1,2} & \cdots & -hk_{1,n} & -\frac{h^2}{24}J_{1,n+\frac{1}{2}} & \frac{h^2}{24}k_{1,\frac{1}{2}} & -\frac{h^2}{24}k_{1,n+\frac{1}{2}} \\
\frac{h^2}{24}J_{2,\frac{1}{2}} & -hk_{2,1} & 1-hk_{2,2} & \cdots & -hk_{2,n} & -\frac{h^2}{24}J_{2,n+\frac{1}{2}} & \frac{h^2}{24}k_{2,\frac{1}{2}} & -\frac{h^2}{24}k_{2,n+\frac{1}{2}} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\frac{h^2}{24}J_{n,\frac{1}{2}} & -hk_{n,1} & -hk_{n,2} & \cdots & 1-hk_{n,n} & -\frac{h^2}{24}J_{n,n+\frac{1}{2}} & \frac{h^2}{24}k_{n,\frac{1}{2}} & -\frac{h^2}{24}k_{n,n+\frac{1}{2}} \\
\frac{h^2}{24}J_{n+\frac{1}{2},\frac{1}{2}} & -hk_{n+\frac{1}{2},1} & -hk_{n+\frac{1}{2},2} & \cdots & -hk_{n+\frac{1}{2},n} & 1-\frac{h^2}{24}J_{n+\frac{1}{2},n+\frac{1}{2}} & \frac{h^2}{24}k_{n+\frac{1}{2},\frac{1}{2}} & -\frac{h^2}{24}k_{n+\frac{1}{2},n+\frac{1}{2}} \\
\hline
\frac{h^2}{24}L_{\frac{1}{2},\frac{1}{2}} & -hH_{\frac{1}{2},1} & -hH_{\frac{1}{2},2} & \cdots & -hH_{\frac{1}{2},n} & -\frac{h^2}{24}L_{\frac{1}{2},n+\frac{1}{2}} & 1+\frac{h^2}{24}H_{\frac{1}{2},\frac{1}{2}} & -\frac{h^2}{24}H_{\frac{1}{2},n+\frac{1}{2}} \\
\frac{h^2}{24}L_{n+\frac{1}{2},\frac{1}{2}} & -hH_{n+\frac{1}{2},1} & -hH_{n+\frac{1}{2},2} & \cdots & -hH_{n+\frac{1}{2},n} & -\frac{h^2}{24}L_{n+\frac{1}{2},n+\frac{1}{2}} & \frac{h^2}{24}H_{n+\frac{1}{2},\frac{1}{2}} & 1-\frac{h^2}{24}H_{n+\frac{1}{2},n+\frac{1}{2}}
\end{array}\right]$$

$$U=\left[\begin{array}{c} u_{\frac{1}{2}} \\ u_1 \\ u_2 \\ \vdots \\ u_n \\ u_{n+\frac{1}{2}} \\ \hline u'_{\frac{1}{2}} \\ u'_{n+\frac{1}{2}} \end{array}\right], \quad F=\left[\begin{array}{c} f_{\frac{1}{2}} \\ f_1 \\ f_2 \\ \vdots \\ f_n \\ f_{n+\frac{1}{2}} \\ \hline f'_{\frac{1}{2}} \\ f'_{n+\frac{1}{2}} \end{array}\right]$$

By solving the above system by any suitable method the numerical solutions of eq.(2.1) is obtained.

Case 2: The partial derivatives $L(x,y)=\frac{\partial^2 k(x,y)}{\partial x \partial y}$ does not exist. In this case, we approximate the integral part that appeared in the right hand side of eq. (2.4) by the composed mid-pointrule to get,

$$u'(x)=f'(x)+h\sum_{j=1}^{n}H(x,x_j)u(x_j) \qquad \text{……..… (2.8)}$$

By setting $x=x_{1/2},\ x_{n+1/2}$ in eq.(2.8), one can get:

$$u'_i=f_i'+h\sum_{j=1}^{n}H_{i,j}u_j\,,\ i=1/2,n+1/2. \qquad \text{…………..… (2.9)}$$

From eq.(2.9) and eq.(2.3) we get the following linear system

$BU = F$ ………..…. (2.10)

of (n+4) equations in (n+4) unknowns $\{u_i\}$, $\{u'_{1/2}\}$ and $\{u'_{n+1/2}\}$, $i=1/2,1,2,...,n,n+1/2$, where the matrices U and F are defined perversely, and

$$B=\left[\begin{array}{cccccc|cc}
1+\frac{h^2}{24}J_{\frac12,\frac12} & -hk_{\frac12,1} & -hk_{\frac12,2} & \cdots & -hk_{\frac12,n} & -\frac{h^2}{24}J_{\frac12,n+\frac12} & \frac{h^2}{24}k_{\frac12,\frac12} & -\frac{h^2}{24}k_{\frac12,n+\frac12}\\
\frac{h^2}{24}J_{1,\frac12} & 1-hk_{1,1} & -hk_{1,2} & \cdots & -hk_{1,n} & -\frac{h^2}{24}J_{1,n+\frac12} & \frac{h^2}{24}k_{1,\frac12} & -\frac{h^2}{24}k_{1,n+\frac12}\\
\frac{h^2}{24}J_{2,\frac12} & -hk_{2,1} & 1-hk_{2,2} & \cdots & -hk_{2,n} & -\frac{h^2}{24}J_{2,n+\frac12} & \frac{h^2}{24}k_{2,\frac12} & -\frac{h^2}{24}k_{2,n+\frac12}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
\frac{h^2}{24}J_{n,\frac12} & -hk_{n,1} & -hk_{n,2} & \cdots & 1-hk_{n,n} & -\frac{h^2}{24}J_{n,n+\frac12} & \frac{h^2}{24}k_{n,\frac12} & -\frac{h^2}{24}k_{n,n+\frac12}\\
\frac{h^2}{24}J_{n+\frac12,\frac12} & -hk_{n+\frac12,1} & -hk_{n+\frac12,2} & \cdots & -hk_{n+\frac12,n} & 1-\frac{h^2}{24}J_{n+\frac12,n+\frac12} & \frac{h^2}{24}k_{n+\frac12,\frac12} & -\frac{h^2}{24}k_{n+\frac12,n+\frac12}\\
\hline
0 & -hH_{\frac12,1} & -hH_{\frac12,2} & \cdots & -hH_{\frac12,n} & 0 & 1 & 0\\
0 & -hH_{n+\frac12,1} & -hH_{n+\frac12,2} & \cdots & -hH_{n+\frac12,n} & 0 & 0 & 1
\end{array}\right]$$

By solving the system given by eq. (2.10) by any suitable method, the numerical solutions of eq.(2.1), is obtained.

3-Numerical examples:

In this section we represent four examples and their numerical results to show the high accuracy of the solution of the linear Fredholm integral equations of the second kind obtained by modified mid-pointmethod and then we compare all results with numerical results obtained by trapezoidal, Simpson, modified trapezoidal and mid-pointmethods, the computations were carried out using MALAB® 7.6.

Example 1 Consider the following linear Fredholm integral equation

$$u(x)=1+\int_0^{\pi}\sin(x+y)u(y)dy,\ \ 0\le x\le\pi \qquad \text{………..…. (3.1)}$$

where the exact solution $u(x)=1+\dfrac{2\cos x+\pi\sin x}{1-\frac{1}{4}\pi^2}$.

It is clear that, $\dfrac{\partial^2 k(x,y)}{\partial x\partial y}=-\sin(x+y)$ exists for $x,y\in[0,\pi]$. Therefore the numerical solution of eq.(3.1) can be obtained by system (2.7), the results errors of which being presented in table 1 for $h=p/10$ and in table 2 for $h=p/20$.

Table 1: Comparison between the results errors via trapezoidal, Simpson's 1/3, Modified trapezoidal, mid-pointand modified mid-pointmethods for example 1 with h= $p/10$.

x_i	trapezoidal method	Simpson's method	modified trapezoidal method	x_i	mid-point method	Modified mid-point method
0	1.1228E-002	7.4634E-005	1.8483E-005	0		1.6177E-005
p/10	1.6129E-002	1.0721E-004	2.6550E-005	p/20	6.9331E-003	1.9952E-005
2p/10	1.9451E-002	1.2929E-004	3.2018E-005	3p/20	9.0170E-003	2.5949E-005
3p/10	2.0869E-002	1.3871E-004	3.4352E-005	5p/20	1.0218E-002	2.9406E-005
4p/10	2.0244E-002	1.3456E-004	3.3323E-005	7p/20	1.0419E-002	2.9985E-005
5p/10	1.7637E-002	1.1723E-004	2.9033E-005	9p/20	9.6002E-003	2.7628E-005
6p/10	1.3304E-002	8.8433E-005	2.1900E-005	11p/20	7.8416E-003	2.2567E-005
7p/10	7.6691E-003	5.0976E-005	1.2624E-005	13p/20	5.3153E-003	1.5297E-005
8p/10	1.2831E-003	8.5287E-006	2.1121E-006	15p/20	2.2688E-003	6.5291E-006
9p/10	5.2285E-003	3.4753E-005	8.6066E-006	17p/20	9.9988E-004	2.8775E-006
				19p/20	4.1706E-003	1.2002E-005
p	1.1228E-002	7.4634E-005	1.8483E-005	p		1.6177E-005
Absolute error	0.1443	9.5896E-004	2.3749E-004		0.0668	2.2455E-004
Lest square error	0.0023	1.0214 E -007	6.2642E-009		5.4779E-004	5.0602E-009

Table 2: Comparison between the results errors via trapezoidal, Simpson's 1/3, Modified trapezoidal, mid-pointand modified mid-pointmethods for example 1 with h= $p/20$.

x_i	trapezoidal method	Simpson method	modified trapezoidal method	x_i	mid-point method	Modified mid-point method
0	2.8036E-003	4.6234E-006	1.1531E-006	0		1.0091E-006
p/10	4.0273E-003	6.6414E-006	1.6564E-006	p/40	1.5707E-003	1.1303E-006
2p/10	4.8567E-003	8.0092E-006	1.9976E-006	5p/40	2.1384E-003	1.5388E-006
3p/10	5.2108E-003	8.5931E-006	2.1432E-006	9p/40	2.4968E-003	1.7967E-006
4p/10	5.0547E-003	8.3358E-006	2.0790E-006	13p/40	2.6107E-003	1.8787E-006
5p/10	4.4039E-003	7.2625E-006	1.8114E-006	17p/40	2.4691E-003	1.7768E-006
6p/10	3.3220E-003	5.4783E-006	1.3664E-006	21p/40	2.0858E-003	1.5010E-006
7p/10	1.9149E-003	3.1579E-006	7.8762E-007	25p/40	1.4984E-003	1.0782E-006
8p/10	3.2038E-004	5.2834E-007	1.3177E-007	29p/40	7.6421E-004	5.4993E-007
9p/10	1.3055E-003	2.1529E-006	5.3697E-007	33p/40	4.4733E-005	3.2190E-008
				37p/40	8.4930E-004	6.1116E-007
p	2.8036E-003	4.6234E-006	1.1531E-006	p		1.0091E-006
Absolute error	0.0693	1.1434E-004	2.8517E-005		0.0332	2.5921E-005
Lest square error	2.8041E-004	7.6257E-010	4.7437E-011		6.8179E-005	3.7342E-011

Example 2 Consider the following linear Fredholm integral equations

$$u(x)=e^{2x}-\frac{1}{2+2x}e^{(x^2+2)}+\frac{1}{2+2x}+\int_0^1 e^{x^2y}u(y)dy,\ \ 0\le x\le 1 \qquad \text{………..}\ (4.2)$$

where the exact solution $u(x)=e^{2x}$. It is clear that, $\frac{\partial^2 k(x,y)}{\partial x\partial y}=\left(2x+2x^3y\right)e^{x^2y}$ exist for $x,y\in[0,1]$.

Therefore the numerical solution of eq.(3.2) can be obtained by system (2.7), the results errors of which being presented in table 3 with h=0.1.

Table 3: Comparison between the results errors via trapezoidal, Simpson's 1/3, Modified trapezoidal, mid-pointand modified mid-pointmethods for example 2 with h=0.1.

x_i	trapezoidal method	Simpson method	modified trapezoidal method	x_i	mid-point method	modified mid-point method
0	7.8964E-002	3.5814E-004	9.0113e-005	0		7.8843E-005
0.1	7.9263E-002	3.5938E-004	9.0424e-005	0.05	3.9837E-002	7.8911E-005
0.2	8.0167E-002	3.6306E-004	9.1350e-005	0.15	4.0140E-002	7.9454E-005
0.3	8.1690E-002	3.6909E-004	9.2869e-005	0.25	4.0751E-002	8.0528E-005
0.4	8.3854E-002	3.7727E-004	9.4926e-005	0.35	4.1679E-002	8.2100E-005
0.5	8.6689E-002	3.8716E-004	9.7412e-005	0.45	4.2937E-002	8.4104E-005
0.6	9.0225E-002	3.9789E-004	1.0011e-004	0.55	4.4540E-002	8.6404E-005
0.7	9.4482E-002	4.0788E-004	1.0261e-004	0.65	4.6500E-002	8.8740E-005
0.8	9.9461E-002	4.1416E-004	1.0417e-004	0.75	4.8824E-002	9.0625E-005
0.9	1.0511E-001	4.1134E-004	1.0341e-004	0.85	5.1498E-002	9.1172E-005
				0.95	5.4470E-002	8.8779E-005
1	1.1126E-001	3.8979E-004	9.7885e-005	1		8.5651E-005
Absolute error	0.9912	0.0042	0.0011		0.4512	0.0010
Lest square error	0.0906	1.6350E-006	1.0344E-007		0.0206	8.6139E-008

Example 3 Consider the following linear Fredholm integral equations

$$u(x) = x^3 + 2x + \frac{1}{20}\left(-\frac{26}{15}x - \frac{13}{15}x^2\right) + \int_0^1 .05y(x^2 + 2x)u(y)dy, \quad 0 \le x \le 1 \qquad \text{………… (3.3)}$$

with the exact solution $u(x) = x^3 + 2x$. It is clear that, $\frac{\partial^2 k(x,y)}{\partial x \partial y} = 0.1(x+1)$ exists for $x, y \in [0,1]$. Therefore the numerical solution of eq.(3.3) can be obtained by system (2.7), the results errors of which being presented in table 3 with h=0.1.

Table 4: Comparison between the results errors via trapezoidal, Simpson's 1/3, modified trapezoidal, mid-pointand modified mid-pointmethods for example 2 with h=0.1.

x_i	trapezoidal method	Simpson method	modified trapezoidal method	x_i	mid-point method	modified mid-pointmethod
0	0	0	0	0		0
0.1	7.3348E-005	1.4672E-007	3.6681E-008	0.05	1.7886E-005	1.5666E-008
0.2	1.5368E-004	3.0742E-007	7.6856E-008	0.15	5.6274E-005	4.9290E-008
0.3	2.4100E-004	4.8210E-007	1.2052E-007	0.25	9.8152E-005	8.5972E-008
0.4	3.3531E-004	6.7074E-007	1.6769E-007	0.35	1.4352E-004	1.2571E-007
0.5	4.3660E-004	8.7336E-007	2.1834E-007	0.45	1.9238E-004	1.6850E-007
0.6	5.4487E-004	1.0900E-006	2.7249E-007	0.55	2.4473E-004	2.1436E-007
0.7	6.6013E-004	1.3205E-006	3.3013E-007	0.65	3.0056E-004	2.6326E-007
0.8	7.8238E-004	1.5651E-006	3.9127E-007	0.75	3.5989E-004	3.1523E-007
0.9	9.1161E-004	1.8236E-006	4.5590E-007	0.85	4.2271E-004	3.7025E-007
				0.95	4.8902E-004	4.2833E-007
1	1.0478E-003	2.0961E-006	5.2402E-007	1		4.5852E-007
Absolute error	0.0052	1.0376e-005	2.5939e-006		0.0023	2.4951E-006
Lest square error	3.6639e-006	1.4661e-011	9.1633e-013		٧.٦٨٣٠E-007	7.9968E-013

Example 4 Consider the following linear Fredholm integral equations

$$u(x) = x - \frac{1}{2} - \frac{2}{5}(x+1)^{5/2}x + \frac{2}{7}(x+1)^{7/2} + \frac{4}{35}x^{7/2} + \int_0^1 \left(1-(x+y)^{3/2}\right)u(y)dy, \ \ 0 \le x \le 1 \quad \ldots\ldots\ (3.4)$$

with the exact solution $u(x) = x$. It is clear that, $\frac{\partial^2 k(x,y)}{\partial x \partial y} = \frac{-3}{4}(x+y)^{-1/2}$ does not exist for $x, y \in [0,1]$. Therefore the numerical solution of eq.(3.4) can be obtained by system (2.10), the results errors of which being presented in table 4 with h=0.1.

Table 5: Comparison between the results errors via trapezoidal, Simpson's 1/3, Modified trapezoidal, mid-pointand modified mid-pointmethods for example 4 with h=0.1.

x_i	trapezoidal method	Simpson method	modified trapezoidal method	x_i	mid-point method	Modified mid-point method
0	4.0927E-003	6.9958E-006	5.6489E-006	0		1.6558E-006
0.1	3.9711E-003	3.9427E-006	3.7976E-006	0.05	2.0183E-003	5.6586E-007
0.2	3.7965E-003	2.9024E-006	3.3414E-006	0.15	1.9424E-003	1.3385E-007
0.3	3.5807E-003	2.2828E-006	2.9849E-006	0.25	1.8439E-003	1.2565E-008
0.4	3.3292E-003	1.8168E-006	2.6396E-006	0.35	1.7265E-003	8.5904E-008
0.5	3.0459E-003	1.4200E-006	2.2843E-006	0.45	1.5923E-003	1.2727E-007
0.6	2.7333E-003	1.0574E-006	1.9119E-006	0.55	1.4430E-003	1.5085E-007
0.7	2.3938E-003	7.1156E-007	1.5198E-006	0.65	1.2798E-003	1.6305E-007
0.8	2.0290E-003	3.7325E-007	1.1072E-006	0.75	1.1035E-003	1.6724E-007
0.9	1.6402E-003	3.6956E-008	6.7405E-007	0.85	9.1492E-004	1.6543E-007
				0.95	7.1476E-004	1.5887E-007
1	1.2289E-003	3.0071E-007	2.2068E-007	1		1.5409E-007
Absolute error	0.0318	2.1840E-005	2.6130E-005		0.0146	3.5407E-006
Lest square error	1.0163E-004	8.5293E-011	8.6286E-011		2.3049E-005	3.2571E-012

5- Conclusions and Recommendations:

In this paper we proposed a numerical method for solving linear Fredholm integral equations and we introduced the modified mid-pointmethod. The system (2.7) solves eq.(1.1) more accurately than system (2.10). Because in system (2.7) we use composed modified mid-pointmethod for solving eq.(2.4) instead composed mid-pointmethod. From numerical examples it can be seen that the proposed numerical method is more accurate than methods such as the trapezoidal, the Simpson, the modified trapezoidal and the mid-pointmethods to estimate the solution of linear Fredholm integral equations when the functions $\frac{\partial k(x,y)}{\partial x}$, $\frac{\partial k(x,y)}{\partial y}$ and $f'(x)$ exist for $x, y \in [a,b]$, also we show that, when the values of h decreases, the errors decrease to small values. We will use this method to study other types of integral equations and their systems in our future work.

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مقارنة بين طريقة النقطة الوسطية المعدلة وبعض الطرق العددية لحل معادلات فريدهولم التكاملية الخطية من النوع الثاني

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المستخلص:

في هذه البحث إستعملنا طريقة النقطة الوسطية المعدّلة لحلّ معادلات فريدهولم الخطيّة التكاملية من النوع الثاني. في الأمثلة العددية بينا فعالية هذه الطريقة، مقارنة بالطرق العددية الأخرى مثل، طريقة النقطة الوسطية، طريقة شبه المنحرف، طريقة سيمبسون وطريقة شبة المنحرف المعدّلة.