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Certain Types of Compact Spaces

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Abstract

In this paper , weusedthe concept of generalized closed (g-closed) and generalized compact (g-compact) sets to constructa new types of compact spaces and functions which are compactly generalized closed space (cgc-space) , generalized compactly generalized closed space and generalized coercive function (g-coercive) and investigate the properties of these concepts .

Keywords: g-open , g-closed , cgc-space , gcgc-space andg-coercive function .

Mathematics Subject Classification:54C10-E45

Introduction

This concept of generalized closed (g-closed) set was introduced by Levin N. [1] and studiedits properties. Selvarani S. [2] gave the definition of g-neighborhood of a point$x \in X$,gT_2 -spaceand g-compact space . The generalized closure of$A \subseteq X$ is the intersection of all g-closed setswhich contain $A$and denoted by$gcl(A)$[1] . In[4] Balachandran K. , Sundaram P. andMaki H. introducedthe certain types of continuous functions. Finally in [3] Ali J. H. and Mohammed J. A. defined certain type of compact functions.We use$T_{\text{ind}}$to denote the indiscrete topology on a non-empty sets$X$ and $T_U$to denote the usual topology on the set of real numbers R. Throughoutthis paper$(X, T)$and(Y, T)(or simply X and Y) represent to non-empty topological spaces on which no separationaxiom are assumed , unless otherwise mentioned .

1. Basic Definitions and Notations:

1.1. Definition [1]:

A subset $A$of a topological space $X$is called generalized closed (for brief g-closed)set if $cl(A) \subseteq U$for everyopen set U in$X$ contains A . The complement of g-closed set is called g-open set.

1.2. Example:

Let $X = \{1, 2, 3\}$ with$T = T_{\text{ind}}$, then $A = \{1\}$ is g-closed set.

1.3. Example:

Let$X = R$, $T = T_U$, then $A = (a, b)$ is not g-closed set.

1.4. Remark [1]:

(i) Every closed set is g-closed.
(ii) Every open set is g-open.

The converse of (i , ii) in remark (1.4) is not true in general as the following example shows:

1.5. Example:

In example (1.2), $A = \{1\}$is g-closed set but not closed and $B = \{2,3\}$is g-open but its not open .

1.6. Theorem [1]:

A subset$A$of a topological space $X$is g-closed set if and only if$cl(A)$-A contains no non-empty closed set.

1.7. Theorem [1]:

A subset $A$of a topological space $X$is g-open if and only if$F \subseteq int(A)$, for every closed set F in Xcontained in A.

1.8. Theorem [1]:

Let X be a topological space, $Y$is a closed (open) set in X. Then:
(i) If B is g-closed (g-open) set in X then $B \cap Y$is g-closed (g-open) set in X.
(ii) If B is g-closed (g-open) set in X then $B \cap Y$is g-closed (g-open) set in$Y$.

1.9. Theorem [1]:

Let X be a topological space and $B \subseteq Y \subseteq X$. Then:
(i) if B is g-closed (g-open) set in Y and Y is g-closed (g-open) set in X, then B is g-closed (g-open) set in X .

(ii) if B is g-closed (g-open) set in X then B is g-closed (g-open) in Y .

Note that if B is g-closed (g-open) in Y then B not necessary be g-closed (g-open) set in X as the following example shows:

1.10. Example:

Let $X = R$ with $T = T_U$ and $Y = \{1 , 2\}$, then $B = \{1\}$ is g-open set in Y, but B is not g-open in R .

1.11. Definition [2]:

Let X be a topological space and $A \subseteq X$. A generalized neighborhood of A(for brief g-neighborhood) is any subset of X which contains g-open set containing A .The family of all g-neighborhoods of a subset A of X denoted by $\mathcal{N}_g(A)$ and the family of all g-neighborhoods of $x \in X$ denoted by $\mathcal{N}_g(x)$.

1.12. Definition [3]:

A topological space X is called generalized Hausdorff (for brief gT_2) if for any two distinct points $x, y \in X$ there are disjoint g-open sets U , V of X such that $x \in U$ and $y \in V$.

1.13. Remark [3]:

Every T_2-space is gT_2-space. But the converse is not true in general. In example (1.2), X is gT_2-space. But X is not T_2-space.

1.14. Remark [2]:

The intersection of two g-closed sets need not be g-closed and the union of two g-open sets need not be g-open as the following example shows:

1.15. Example:

Let $X = \{a , b , c\}$ and $T = \{\emptyset , X , \{a\}\}$ be a topology on X , then $\{a , b\}$ and $\{a , c\}$ are g-closed sets in X, but $\{a , b\} \cap \{a , c\} = \{a\}$ is not g-closed set and $\{b\}, \{c\}$ are g-open sets but $\{b\} \cup \{c\} = \{b, c\}$ is not g-open .

1.16. Definition [2]:

A topological space X is called generalized multiplicative space (IG-space) if arbitrary intersection of g-closed sets of X is g-closed set .

1.17. Remark [2]:

(i) $gcl(A)$ need not be g-closed, since the intersection of g-closed sets is not to be g-closed .
(ii) $x \in gcl(A)$ if and only if for every g-open set U containing x , $U \cap A \neq \emptyset$.
(iii) If X be an IG-space, then $gcl(A)$ is g-closed set.
(iv) Every T_1-space is an IG-space.

1.18. Definition [4]: Let $f: X \to Y$ be a function from a topological space X into a topological space Y, then f is called:

(i) generalized continuous (g-continuous) function if $f^{-1}(A)$ is g-closed set in X for every closed set A in Y .

(ii) generalized irresolute continuous (gI-continuous) function if $f^{-1}(A)$ is g-closed set in X for every g-closed set A in Y .

1.19. Definition [4]:

A function $f: X \to Y$ is called:

(i) generalized closed (g-closed) if $f(B)$ is g-closed set in Y for every closed set B in X.

(ii) generalized irresolute closed (gI-closed) function if $f(B)$ is g-closed set in Y for every g-closed set B in X .

1.20. Definition [4]:

A function $f: X \to Y$ is called:

(i) generalized open (g-open) function if $f(B)$ is g-open set in Y for every open set B in X.

(ii) generalized irresolute open (gI-open) function if $f(B)$ is g-open set in Y for every g-open set B in X .

1.21. Definition [3]:

A topological space X is called generalized compact (g-compact) space if every g-open cover of X has finite subcover.

1.22. Remark [5]:

Every g-compact space is compact. The converse is not true in general as the following example shows:

1.23. Example [5]:

Let $X = \{x\} \cup \{ x_i : i \in I \}$, I uncountable, $T = \{\emptyset, X , \{x\}\}$ be a topology on X . Then X is compact but is not g-compact, since $\{\{x , x_i\} : i \in I\}$ is g-open cover of X and has no finite subcover.

1.24. Theorem [2] ,[3],[5]:

(i) Every g-closed subset of g-compact space is g-compact.
(ii) The intersection of g-compact subset with g-closed subset is g-compact.
(iii) Every g-compact subspace of gT_2-space is g-closed.
(iv) Every finite subset is g-compact.
(v) Every T_1 compact space is g-compact.

1.25. Theorem [3]:

(i) Let X be a topological space and F is g-closed subset of X. Then $F \cap K$ is g-compact in F for every g-compact set K in X .

(ii) Let Y be a g-open set of a topological X and $K \subseteq Y$, then K is g-compact set in Y if and only if K is g-compact set in X .

1.26. Theorem [3]:

(i) Let $f$be gI-continuous function from g-compact space X onto a topological space Y, then $Y$is g-compact space.
(ii) Let $f: X \to Y$be gI-continuous function, then the image $f(A)$ of any g-compact set A in X is g-compact set in$Y$.
(iii) Let $f$be gI-continuous function from g-compact space Xinto agT_2-space$Y$ is gI-closed.

1.27. Definition [3]:

Let $f: X \to Y$be a function, then$f$ is calledgeneralized irresolute compact (gI-compact) if$f^{-1}(K)$is g-compact set in $X$for every g-compact set K in$Y$.

1.28. Definition [6]:

A set $D$is called a directed if there is a relation $\leq$ on Dsatisfying:
(i)$d \leq d$ for each $d \in D$.
(ii) If $d_1 \leq d_2$ and $d_2 \leq d_3$ then$d_1 \leq d_3$.
(iii) If $d_1, d_2 \in D$, there is some$d_3 \in D$ with $d_1 \leq d_3$ and$d_2 \leq d_3$.

1.29. Definition [7]:

A netin a set Xisa function$\chi: D \to X$, where$D$ is directed set. The point$\chi(d)$ is usually denoted by χ_d.

1.30. Definition [7]:

A subnet of a net $\chi: D \to X$is the composition $\chi o \varphi$,where$\varphi: M \to D$ and M is directed set, such that :
(i)$\varphi(m_1) \leq \varphi(m_2)$, where $m_1 \leq m_2$.
(ii) For all$d \in D$there is some $m \in M$ such that$d \leq \varphi(m)$for$m \in M$. Thepoint $\chi o \varphi(m)$ is often writtenχ_{dm} .

1.31. Definition [7]:

Let $(\chi_d)_{d \in D}$ be a net in a topological space $X$and $A \subseteq X$,$x \in X$ then:
(i) $(\chi_d)_{d \in D}$is eventually in A if there is $d_0 \in D$such that $\chi_d \in A$for all$d \geq d_0$.
(ii) $(\chi_d)_{d \in D}$is frequently in$A$if for all$d \in D$ there is$d_0 \in D$with$d \geq d_0$such that $\chi_{d_0} \in A$.

1.32. Definition [5]:

Let $(\chi_d)_{d \in D}$be a net in a topological space X, $x \in X$. Then $(\chi_d)_{d \in D}$is said to be generalized converges to a point x (for brief g-converges) if $(\chi_d)_{d \in D}$eventually in every g-neighborhood of x (written$\chi_d \xrightarrow{g} x$) . A point x is called generalized limit point (for brief g-limit point) of $(\chi_d)_{d \in D}$.

1.33. Theorem:

Let$X$be a topological space and $A \subseteq X$, $x \in X$. Then $x \in gcl(A)$if and only if there is a net $(\chi_d)_{d \in D}$in A such that $\chi_d \xrightarrow{g} x$.

Proof:

Suppose that there is a net$(\chi_d)_{d\in D}$in A such that $\chi_d \overset{g}{\to} x$. To prove that $x \in gcl(A)$.
Let$U \in \mathcal{N}_g(x)$, since $\chi_d \overset{g}{\to} x$, there is $d_0 \in D$ with$\chi_d \in U$for all $d \geq d_0$. But$\chi_d \in U$ for all $d \in D$. So $A \cap U \neq \emptyset$ for all $U \in \mathcal{N}_g(x)$. By remark (1.17.ii) , $x \in gcl(A)$.

Conversely:

Suppose that $x \in gcl(A)$. To prove that there is a net $(\chi_d)_{d\in D}$in A such that $\chi_d \overset{g}{\to} x$.
Since$x \in gcl(A)$, by remark (1.17.ii) , $A \cap U \neq \emptyset$for all $U \in \mathcal{N}_g(x)$. Then$D = \mathcal{N}_g(x)$is directed set by inclusion. Since $\cap U \neq \emptyset \forall U \in \mathcal{N}_g(x)$, there is $\chi_U \in A \cap U$. Define $\chi: D \to A$ by$\chi(U) = \chi_U$ for all$U \in \mathcal{N}_g(x)$. Hence $(\chi_U)_{U\in \mathcal{N}_g(x)}$is a net in$A$. To prove that$\chi_U \overset{g}{\to} x$.
Let $U \in \mathcal{N}_g(x)$to find $d_0 \in D$such that $\chi_d \in U$ for all$d \geq d_0$. Let $d_0 = U$, then for all $d \geq d_0$ we have $d = V \in \mathcal{N}_g(x)$ i.e., $V \geq U \Leftrightarrow V \subseteq U$.
$\chi_d = \chi(d) = \chi(V) = \chi_V \in V \cap A \subseteq V \subseteq U$, then $\chi_V \in U$for all $d \geq d_0$. Thus$\chi_U \overset{g}{\to} x$.

1.34. Corollary:

Let Xbe a topological space and $A \subseteq X$, $x \in X$. Then $x \in gcl(A)$if and only if thereis a net $(\chi_d)_{d\in D}$in A such that $\chi_d \overset{g}{\propto} x$.

1.35. Theorem [8]:

Let$X$ be aT_2-space. Then $X$is g-compact if and only if every net in X has a g-cluster point in X.

1.36. Remark [7]:

Let $f: X \to Y$be a function from a set Xinto a set$Y$, then:

(i) If $(\chi_d)_{d\in D}$is a net in X, then $\{f(\chi_d)\}_{d\in D}$is a net in Y .

(ii) If$f$is onto and$(y_d)_{d\in D}$be a net in Y , then there is a net $(\chi_d)_{d\in D}$in$X$such that $f(\chi_d) = y_d$, for each$d \in D$.

1.37. Theorem:

Let $X$and Ybe topological spaces . A function $f: X \to Y$ is g-continuous if and only if whenever $(\chi_d)_{d\in D}$is a net in X such that $\chi_d \overset{g}{\to} x$, then $f(\chi_d) \to f(x)$ in$Y$.

Proof: Clear.

1.38. Corollary:

Let $X$and Ybe topological spaces . A function $f: X \to Y$ is gI-continuous if and only if whenever $(\chi_d)_{d\in D}$is a net in X such that $\chi_d \overset{g}{\to} x$, then $f(\chi_d) \overset{g}{\to} f(x)$ in$Y$.

Proof:

Suppose that $f: X \to Y$ is gI-continuous and $(\chi_d)_{d\in D}$ is a net in X such that $\chi_d \overset{g}{\to} x$. To prove that $f(\chi_d) \overset{g}{\to} f(x)$. Let $V \in \mathcal{N}_g(f(x))$ in Y, then $f^{-1}(V) \in \mathcal{N}_g(x)$, for some $d_0 \in D$, $d \geq d_0$ implies that $\chi_d \in f^{-1}(V)$. Thus showing that $f(\chi_d) \overset{g}{\to} f(x)$, since $(\chi_d)_{d\in D}$ is eventually in each g-neighborhood of $f(x)$, then by remark (1.36.i), $\{f(\chi_d)\}$ is a net in Y which is eventually in each g-neighborhood of $f(x)$. Therefore $f(\chi_d) \overset{g}{\to} f(x)$.

Conversely:

To prove that f is gI-continuous,suppose not, then there is $V \in \mathcal{N}_g(f(x))$ such that $f(U) \not\subset V$ for any $U \in \mathcal{N}_g(x)$. Thus for all $U \in \mathcal{N}_g(x)$ we can $\chi_U \in U$ such that $f(\chi_U) \notin V$, but $(\chi_U)_{U\in\mathcal{N}_g(x)}$ is a net in X with $\chi_U \overset{g}{\to} x$, while $\{f(\chi_U)\}_{U\in\mathcal{N}_g(x)}$ is not g-convergent to $f(x)$. This is a contradiction.

2. Compactly g-closed and g-compactly g-closed spaces:

This section is devoted to a new concept which is called compactly g-closed space and generalized compactly g-closed space. Several various examples , theorems and remarks on these concepts are proved . Furthermore theorems are stated as well as the relationships between these concepts.

2.1. Definition:

Let X be a topological space. A subset $A \subseteq X$ is called compactly generalized closed (for brief cgc-set) if $A \cap K$ is g-compact set for every g-compact set K in X.

2.2. Example:

(i) Every finite subset of a topological space is cgc-set.
(ii) Every subset of indiscrete space is cgc-set.

2.3. Theorem:

Every g-closed subset of a topological space is cgc-set.

Proof:

Let A be a g-closed subset of a topological space X and K be a compact subset of X, by Theorem(1.24.ii) , $A \cap K$ is g-compact set. Thus A is cgc-set.

The converse of theorem (2.3) need not true in general as the following example shows:

2.4. Example:

Let $X = \{a , b, c\}$ and $T = \{\emptyset, X, \{a, b\}\}$ be a topology on X , then $A = \{a, b\}$ is cgc-set but it is not g-closed set.

2.5. Theorem:

Let X be a T_2-space and$A \subseteq X$. Then$A$is cgc-set if and only if it is g-closed set.

Proof:

Let A be a cgc-set in X and $x \in gcl(A)$. By theorem (1.33), there is a net $(\chi_d)_{d\in D}$in A such that$\chi_d \overset{g}{\to} x$. Then$K = \{\chi_d, x\}$is g-compact set . Since A is cgc-set , then $A\cap K$ is g-compact set in$X$. But X is a T_2 , then $A\cap K$is g-closed . Since $\chi_d \overset{g}{\to} x$and $\chi_d \in A\cap K$, then by theorem (1.33) , $x \in A\cap K$, hence$x \in A$. Thus A is g-closed set.
Conversely: By using Theorem (2.3) .

2.6. Theorem:

Let$f: X \to Y$is a bijective, gI-continuous , gI-compact function and$A \subseteq X$. Then A is cgc-set in X if and only if $f(A)$ is cgc-set in$Y$.

Proof:

Let$A$ be a cgc-set in$X$ and let Kbe a g-compact set in$Y$. Since f be a gI-compact, then $f^{-1}(K)$is g-compact set in$X$. Thus$A\cap f^{-1}(K)$is g-compact set in$X$. By theorem (1.26.ii),$f\left(A\cap f^{-1}(K)\right)$is g-compact set in$Y$. But $f\left(A\cap f^{-1}(K)\right) = f(A)\cap K$is g-compact set in Y.
Hence $f(A)$is cgc-set in Y.
Conversely:
Let$f(A)$be a cgc-set in$Y$. To prove that$A$ is cgc-set in . Let$K$ be a g-compact set in$X$. Since $f$be a gI-continuous , then by theorem (1.26.ii), $f(K)$is g-compact set in$Y$ .Thus $f(A)\cap f(K)$ is g-compact set in$Y$,thus$f^{-1}\left(f(A)\cap f(K)\right)$is g-compact set in$X$.(since f gI-compact). But$f^{-1}\left(f(A)\cap f(K)\right) = A\cap K$. Thus$A$ is cgc-set in$X$.

2.7. Theorem:

Let Bbe a g-open subset of a topological space X. Then$B$is cgc-set in Xif and only if the inclusion function$i: B \to X$is gI-compact.

Proof:

Suppose that B be a cgc-set and K be a g-compact set in X . Then$B\cap K$is g-compact set in$X$, by theorem (1.25.ii), $B\cap K$ is g-compact set in B. But $B\cap K = i^{-1}(K)$, then $i^{-1}(K)$ is g-compact set in B . Thus $i: B \to X$ is gI-compact.
Conversely:
Let K be a g-compact set in$X$, since$i: B \to X$is gI-compact . Then$i^{-1}(K) = B\cap K$ is g-compact set in$B$, thus by theorem (1.25.ii),$B\cap K$is g-compact set in $X$for every g-compact set K in X , Therefore B is cgc-set in X.

2.8. Definition:

A subset $A$of a topological space$X$ is said to be generalized compact generalized closed set (for brief gcgc-set) , if $A\cap K$is g-closed set in X for every g-compact set Kin$X$.

2.9. Example:

Every subset of a discrete space is gcgc-set.

2.10. Remark:

Not every set of a topological space is gcgc-set as the example (2.4) shows.

2.11. Theorem:

Every gcgc-set in a topological space is gcg-set.

Proof:

Let A be a gcgc-set of a topological space X and let K be a g-compact subset of X. Then $A\cap K$ is g-closed set in X . Since $A\cap K \subseteq K$, then by remark (1.24.i) , $A\cap K$ is g-compact set . Therefore A is cgc-set in X.

2.12. Theorem:

Let X be a T_2-space and $A \subseteq X$, the following statements are equivalent:

(i) A is cgc-set.

(ii) A is gcgc-set.

(iii) A is g-closed set.

Proof:

(**i** $\Longrightarrow$ **ii**) Let A is cgc-set in X and let K be a g-compact set in X. Then $A\cap K$ is g-compact set in X. Since X is a T_2-space , then by theorem (1.24.iii) , $A\cap K$ is g-closed set in X. Thus A is gcgc-set in X .

(**ii** $\Rightarrow$ **i**) By using theorem (2.11) .

(**iii** $\Rightarrow$ **i**) By using theorem (2.3) .

2.13. Remark:

If X is not T_2-space , then it is not necessary that cgc-set is gcgc-set as the following example shows:

Let $X = \{a, b, c\}$ and $T = \{ U \subseteq X : a \in U \}\cup\{\emptyset\}$ be a topology on X, clear that (X, T) is not T_2-space . Since $\{a, b\}, \{b\} \subset X$ and $\{b\}$ is g-compact set in X and $\{a, b\}\cap\{b\} = \{b\}$ is g-closed but $\{a, b\}$ is not g-closed set.

Recall that a bijective function $f: X \to Y$ is called generalized irresolute homeomorphism (gI-homeomorphism) if f and f^{-1} are gI-continuous [7].

2.14. Theorem [9]:

A bijection function $f: X \to Y$ is gI-homeomorphism if f is gI-continuous and gI-open (gI-closed) function.

2.15. Theorem:

The following conditions on a Hausdorff space $Y$are equivalent:
(i) The only g-open subset of$Y$which is gcgc-set is the whole spaceand the empty set.

(ii) Every gI-open , gI-continuous and gI-compact function from a topologicalspace Xinto Y is onto .

(iii) Every one to one, gI-open , gI-continuous and gI-compact function froma topological space$X$ into $Y$is gI-homeomorphism.

Proof:

(**i** ⇒ **ii**)Let $f: X \to Y$ be a gI-open , gI-continuous and gI-compact function .Since$X$isnon - empty g-open set , then $f(X)$is non-empty g-open set in Y. To prove $f(X)$is gcgc-set in Y. Let Kbe a g-compact set in Ythen$f^{-1}(K)$is g-compact set in$X$, since$f$ is gI-compact. Thus by theorem (1.26.ii) , $f(f^{-1}(K))$ is g-compact set in Y. By theorem (1.24.iii) ,$f(f^{-1}(K))$is g-closed set in Y. Since $f(X) \cap K = f\left(f^{-1}(K)\right)$,then $f(X) \cap K$is g-closed set in Y. So $f(X)$is gcgc-set.But$f(X) \neq \emptyset$, then$f(X) = Y$. Thus f is onto.

(**ii** ⇒ **iii**)Let $f: X \to Y$ be an one to one, gI-open, gI-continuous and gI-compact function. Then by (ii), f is onto and one to one ,hence it is bijection . Then by theorem (2.14) ,f is gI-homeomorphism .

(**iii** ⇒ **i**)Let A be a non-empty g-open subset of Ywhich is gcgc-set. Then by theorem (2.11) , $A$is cgc-set, since $A$is g-open . Then by theorem (2.7) , theinclusion function $i: A \to Y$is gI-compact. To prove $i: A \to Y$ is gI-continuous, let $B$is g-open set in Y, then $A \cap B$is g-open set. But$A \cap B = i^{-1}(B)$isg-open set in A. Thus , the inclusion function is gI-continuous, by (iii) , the inclusion function is gI-homeomorphism . Thus$A = Y$, this complete proof .

2.16. Definition:

A topological space $X$is said to be compactly generalized closed space (for brief cgc-space) if every cgc-set of X is g-closed.

2.17. Example:

(i) Every indiscrete space is cgc-space.
(ii) Every T_2-space is cgc-space.

2.18. Remark:

The example in remark (2.13) shows that not every topological space is cgc-space.

2.19. Theorem:

Let X be a topological space and$Y$is cgc-space. Then every gI-continuous and gI-compact onto function $f: X \to Y$is gI-closed.

Proof:

Let Fbe a g-closed subset of$X$. To prove that $f(F)$is g-closed subset of$Y$. Let Kbe a g-compact subset of Y. Since$f$ is gI-compact, then$f^{-1}(K)$ isg-compact set in X.
By remark (1.24.ii) ,$F \cap f^{-1}(K)$is g-compact set in$X$.
Since f is gI-continuous, then by theorem (1.26.ii), $f(F \cap f^{-1}(K))$ is g-compact set of Y.
But$f\left(F \cap f^{-1}(K)\right) = f(F) \cap K$, thus$f(F) \cap K$is g-compact set of Y. Hence $f(F)$is cgc-set in Y. Since Y is cgc-space, then $f(F)$is g-closed set in$Y$. Thus f is gI-closed function.

2.20. Definition:

A topological space$X$is said to be generalized compactly generalized closed (for brief gcgc-space) if every gcgc-set of $X$is g-closed.

2.21. Example:

(i) Every T_2-space is gcgc-space.
(ii) Every indiscrete space is gcgc-space.

2.22. Theorem:

Let Xbe a T_2-space . Then cgc-space and gcgc-space are equivalent.

Proof: By using theorem (2.12) .

2.23. Definition:

Let $X$and Ybe topological spaces. A function $f: X \to Y$ is called generalized coercive (for brief g-coercive) if for every g-compact subset $B$of Y there is g-compact subset $A$of$X$ such that $f(X \setminus A) \subseteq (Y \setminus B)$.

2.24. Example:

The identity function of any topological space is g-coercive.

2.25. Theorem:

If$f: X \to Y$ is a function, such that $X$is g-compact space , then f is g-coercive.

Proof:

Let Bbe a g-compact subset of$Y$. Since X is g-compact space.Then $(X \setminus X) = f(\emptyset) = \emptyset \subseteq f(Y \setminus B)$. Thus f is g-coercive function.

2.26. Theorem:

Let X and Y beT_2-spaces and $f: X \to Y$is gI-continuous function. Then $f$is g-coercive if and only if f is gI-compact .

Proof:

Suppose that f is g-coercive and let B be a g-compact subset of Y. To prove that f is g-compact, since Y is T_2-space then by (1.24.iii), B is g-closed but f is gI-continuous . Then $f^{-1}(B)$ is g-closed subset of X. Since f is g-coercive function, then there is a g-compact set A in X such that $f(X \setminus A) \subseteq (Y \setminus B)$.

since $f^{-1}(B)$ is g-closed, then by corollary (1.34) , every net in $f^{-1}(B)$ hasg-cluster in itself itself . Then by theorem (1.35) , $f^{-1}(B)$ is g-compact subset in X. Therefore f is gI-compactfunction.

Conversely: By using theorem (2.25).

2.27. Theorem:

Let X and Y betopological spacesand $f: X \to Y$ be a function. Then:

(i) If $f: X \to Y$ be a g-coercive function with F is g-closed and open subset of X, then the restriction function $f_{/F}: F \to Y$ is g-coercive .

(ii) If X is g-compact and F is g-closed subset of X, then $f_{/F}: F \to Y$ isg-coercivefunction.

Proof:

(i) Let B be a g-compact subset of Y , since f is a g-coercive . Then there is a g-compact subset A of X such that $f(X \setminus A) \subseteq (Y \setminus B)$.

Since F isg-closed subset of X , then by theorem (1.24.ii), $F \cap A$ is g-compact set in X. Since F is open in X ,by theorem (1.25.ii), $F \cap A$ is g-compact set in F.

Since $f_{/F}(F \cap A) = f(F \setminus A)$ and $F \setminus A \subseteq X \setminus A$, then $f(F \setminus A) \subseteq f(X \setminus A)$.

Thus $f_{/F}(F \setminus F \cap A) \subseteq Y \setminus B$, hence $f_{/F}: F \to Y$ is g-coercive.

(ii) By using theorem (2.25) and (i) .

2.28. Theorem:

A composition of two g-coercive functions is g-coercive.

Proof:

Let $f: X \to Y$ and $h: Y \to Z$ be a g-coercive functions . Let C is a g-compact subset of Z , then there is a g-compact subset B of Y such that $h(Y \setminus B) \subseteq Z \setminus C$.

Since f is a g-coercive, then there is a g-compact subset A of X such that $(X \setminus A) \subseteq Y \setminus B$.

So $h(f(X \setminus A)) \subseteq h(Y \setminus B)$, but $h(Y \setminus B) \subseteq Z \setminus C$ .Hence $h(f(X \setminus A)) = hof(X \setminus A) \subseteq Z \setminus C$, therefore hof is g-coercive function.

2.29. Theorem:

If $f: X \to Y$ is bijective , gI-compact and $h: Y \to Z$ is a g-coercive function, then hof is g-coercive function .

Proof:

Let C be a g-compact subset of Z , then there is a g-compact subset B of Y such that $h(Y \setminus B) \subseteq Z \setminus C$. Put $A = f^{-1}(B)$, since f is gI-compact then A is a g-compact subset of X.Thus $hof(X \setminus A) = h(f(X \cap A^c)) = h(f(X) \cap f(A^c))$.

Since f is a bijective, then $hof(X \setminus A) = h(f(X) \cap f(A^c)) = h(Y \setminus f(f^{-1}(B))^c = h(Y \cap B^c)$ $= h(Y \setminus B) \subseteq Z \setminus C$. Thus hof is g-coercive function.

Reference:

[1] . N. Levine . Generalized closed sets in topology . Rend. Circ. Mat. Palermo, 19(2), 89-96 , (1970) .

[2] . S. Selvarani . Generalization of extreme value theorems in g-compact spaces , generalization of Intermediate value theorem g-connected spaces and generalization of Urysohon's Lemma and Tietze extension theorem in g-multiplicative spaces , International Journal of Computer , Vol. 2 , April , (2013) .

[3] . H. J. Ali and J. Mohammed . On g , g*, g**-compact functions , Journal of Al -Nahrain University Vol. 13(2) , pp. 205-209 , (2010) .

[4] . K. Balahandram , P. Sundaram and H. Maki . On generalized continuous maps in topological spaces , Mem. Fac. Sci. Kochi Univ. Ser. A. Math. , 12, 5-13 , (1991) .

[5] . M. Caldas , S. Jafari , P. Moshokoa and T. Noiri . On Go-Compact Spaces , Factually of Sci. and Math. , Univ. of Nis , Serbia , 22:1, 47-55 , (2008) .

[6] . E. H. Moore and H.L. Smith . A General theory of limits . Amer. J. Math., 44 , 102-121 , (1922) .

[7] . S. Willard . ***General topology*** . Addison -Wesley Publishing Company, Inc. , (1970) .

[8] . M. Caldas, S. Jafari ,M. Latif and O. Ozbakir . GO-Compact and GO-(m , n)- Compact Spaces , Technical Report Series, TR 399, July, (2008) .

[9] . H. Maki , P. Sundaram and H. Maki . On generalized homeomorphisms in topological spaces . Bull. Fukuoka Univ. Part III, 40 , 13-21, (1991) .

[10] .N. Bourbaki Elements of mathematics . ***General topology*** , Spring verlog , Berlin , Heidelberg , New York , London , Paris , Tokyo 2nd , Edition , (1989) .

أنماط معينة من الفضاءات المرصوصة

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المستخلص :
في هذا البحث أستخدمنا مفهومي المجموعات المغلقة المعممة (المغلقة-g) والمرصوصة المعممة (المرصوصة-g) لأنشاء أنواع جديدة من الفضاءات المرصوصة والدوال أسميناهاالفضاءاتالمرصوصة المغلقة المعممة (الفضاءات cgc-)والفضاءات المعممة المرصوصة المعممة (الفضاءات-gcgc) والدوال الأضطرارية المعممة (الأضطرارية-g)ودرسنا خواص هذه المفاهيم .