Applications of the B-SPLINE Surfaces

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Abstract

The paper describes the fundamental notions cognate to the Spline and B-Spline curves and surfaces for the purport of their utilization in applications of obnubilated signs perception. By utilizing B-spline surfaces, one can establish identification algorithms of the obnubilated sources, by designates of their application together with kenned field techniques. Furthermore, these are largely utilized in computerized graphics for modelling and design, as they have many geometrical and calculable properties

1- Initiation

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1.2 FREEFORM CURVES

In projection activities, one is often faced with the indispensability of constructing curves kenned through their forms, and not through equations. Thus, the designer may approximate the form of a curve through a set of points, and predicated on these, a program can calculate all the points indispensable for marking the curve. The Freeform curves are utilized in architecture, artistic design, for animation, modelling, in the form perception systems of images, etc. Free forms is analytically modelled through interpolation and approximation curves and surfaces.

There are more types of Freeform curves, some of which is designated only through 2D or 3D points, while others need the designation of supplemental geometrical restrictions. (Contiguous vectors within the given points.)

There are two geometric modelling ways of the Freeform curves:

O Based on linear interpolation – the resulted curve goes through all given points. The higher the number of the points, the exacter the representation is .

O Based on the ‚smoothing' of the polygon composed of the given points – the resulted curve (called approximation curve) does not go through all the given points, which have the role of describing the form and position of the curve. This is why the given points are called ‚describing points' or ‚control points' of the curve.

Approximation curves are subsidiary as they do not require the exact cognizance of the designed object form. The computerized projection systems of the Freeform curves sanction the interactive control of a curve form through smooth forms of kineticism of the control points.

2.2 PARAMETRIC EQUATIONS OF A CURVE

The form functions $f(x) = y$ cannot be used for the design of the interpolation/ approximation curves, due to the following reasons:

- There may be more points on the curve for a particular value of x ;
- The forms designed through such curves are independent of any coordinate systems and are not determined through the relation between the points and a certain coordinate system, but through the relation of the given points;
- The Freeform curves are often planar, these are not being able to be represented by non-parametric equations.

The design of a 3D curve is obtained from a 2D curve and the addition of the equation that results in the component z of the points on the curve.

The parametric equations introduce an auxiliary variable, u , describing a flat curve by means of two functions, $F_x(u)$ and $F_y(u)$, which define the evolution of the two projections, x and y, based on the parameter u. One point (x, y) belongs to the curve if there is a value u in such a way that $x = F_x(u)$ and $F_y(u)$.

The parametric equations of a line that goes through the points ($x1, y1$), ($x2, y2$) are:

$$x(u) = x1 + u(x2 - x1) = a_{1,x}u + a_{0,x}$$

$$y(u) = y1 + u(y2 - y1) = a_{1,y}u + a_{0,y}$$

For a curve, the polynoms $F_x(u)$ and $F_y(u)$ have a degree bigger than 1:

$$x(u) = a_{n,x}u^n + a_{n-1,x}u^{n-1} + ... + a_{1,x}u + a_{0,x}$$

$$y(u) = a_{n,y}u^n + a_{n-1,y}u^{n-1} + ... + a_{1,y}u + a_{0,y}$$

In the 3D space, a segment of a parametric cubic curve is defined through the following equation system:

$$x(u) = a_{3,x}u^3 + a_{2,x}u^2 + a_{1,x}u + a_{0,x}$$

$$y(u) = a_{3,y}u^3 + a_{2,y}u^2 + a_{1,y}u + a_{0,y}$$
$$0 \le u \le 1$$

$$z(u) = a_{3,z}u^3 + a_{2,z}u^2 + a_{1,z}u + a_{0,z}$$

The 12 coefficients are the algebraic factors of the curves. They determine the form, size and position of the curve. Two curves with identical form that occupy two different positions in space have different algebraic factors. For an easier representation, the vectorial notation is used:

$$p(u) = a_3u^3 + a_2u^2 + a_1u + a_0 \quad ,$$

where a_3, a_2, a_1, a_0 are vectors with three components, and $p(u)$ is the position vector of a point $(x(u), y(u), z(u))$ on the curve.

2.3 SPLINE CURVES

The spline curves are interpolation curves. The word „spline” refers to an instrument utilized in the technical design for drawing the planar curves. The spline is an elastic band fine-tuned at the points through which the curve must be drawn, by denotes of some weights called „ducks”. A spline curve may by drawn through an illimitable number of control points .

Between two control points, the form of the spline band is mathematically represented by a cubic polynomial. Customarily, a spline curve is represented through a k degree integral rational function defined by the components, with differentials of type $k-1$perpetual at the connecting points. Thus, the cubic spline curve has second-order continuity at the connecting points .

Small degree polynomials are being utilized for the representation of spline curves, in order to reduce the calculation of the points on the curve and to evade the numerical inequality. The spline curve is composed of adjacent segments represented by betokens of diminutive degree polynomials (2 or 3), as a minute degree polynomial cannot interpolate a curve through an arbitrary number of points.

A cubic spline curve segment is represented by means of the following parametric equation:

$$p(t) = \sum_{i=1}^{4} B_i * t^{i-1} \; t1 \le t \le t2 \;,$$

Where:

$t1$ and $t2$ are the values of the parametrical differential at the initial and the end point of the curve segment;

$p(t) = [x(t) \quad y(t) \quad z(t)]$ is the position vector of a point on the curve;

$B_i = [B_{ix} \quad B_{iy} \quad B_{iz}]$ is calculated based on 4 boundary conditions.

Assuming P_1, P_2 are the bending points of the curve segment and $P^t{}_1, P^t{}_2$ are the unit tangent vectors at the bending points. Considering $t_1 = 0$ and enforcing the 4 boundary condiresultsthe following resutls:

$$B_1 = P_1$$

$$B_2 = P^t{}_1$$

$$B_3 = 3(P_2 - P_1)/t_2^2 - 2p_1^t / t_2 B_3$$

$$B_4 = 2(P_1 - P_2) ? t_2 \sup 3 + p_1^2 / t_2^2 + p_2^t / t_2^2$$

Generally, a spline curve that goes through a number of n control points is represented by the following equation:

$$p_k(t) = \sum_{i=1}^{4} B_i, k * t^{i-1} \; 0 \le t \le t_{k+1}$$

$,1 \le k \le n-1$

Where:

$$B = \begin{bmatrix} B_{1},_k \\ B_{2},_k \\ B_{3},_k \\ B_{4},_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/t_{k+1}^2 & -2/t_{k+1} & 3/t_{k+1}^2 & -1/t_{k+1} \\ 2/t_{k+1}^3 & 1/t_{k+1}^2 & -2/t_{k+1}^3 & 1/t_{k+1}^2 \end{bmatrix} \begin{bmatrix} p_k \\ p_k^t \\ p_{k+1} \\ p_{k+1}^t \end{bmatrix}$$

A reduction of the calculation is obtained by taking the intervals $t_k - t_{k-1}$ equal to 1 for all the curve segments $0 \le t \le 1$. In this case, the spline curve is described by the folllowing matrix equation:

$$p_k(t) = [F1(t) \quad F2(t) \quad F3(t) \quad F4(t)]\,[P_k \; P_{k+1} \; P^t{}_k \; P^t{}_{k+1}]^T$$

where:

$$F1(t) = 2t^3 - 3t^2 + 1$$

$$F2(t) = -2t^3 + 3t^2$$

$$F3(t) = t^3 - 2t^2 + t$$

$$F4(t) = t^3 - t^2$$

$$[F] = [F1(t) \;\; F3(t) \;\; F4(t)] = [t^3 \; t^2 \; t \; t] \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The following results:

$$p_k(t) = [F]\,[G_k] = [T]\,[N]\,[G_k]$$

The matrices T and N are the same for all the segments of the spline curve, only the matrix G_k changes from one segment to the other.

2.4 B-SPLINE CURVES

The B-spline curves are approximation curves defined by means of control points. They are described by means of polynomial functions defined on portions, which give them the property of local control. The segments of the B-spline curve are desbribed through second or third degree polynoms, this being independent from the number of the control points.

The B-spline curves are determined by the vectorial equation:

$$p(u) = \sum_{i=0}^{n} p_i * N_{i,k}(u)$$

where:

p_i are the control points

$N_{i,k}(u)$ are the mixture functions (B-spline functions)

k establishes the degree of the approximation polynom $(k-1)$ and the continuity order $(k-2)$ of the curve.

The B-spline functions are determined recursively, as follows:

$N_{i,1}(u) = 1$ for $t_i \le u \le t_{i+1}$

$= 0$ otherwise

$$N_{i,k}(u) = \frac{u - t_i}{t_i + k - 1 - t_i} * N_{i,k-1}(u) + \frac{t_{i+1} - u}{t_{i+k} - t_{i+1}} * N_{i+1,k-1}(u)$$

This shows that a function $N_{i,k}(u)$ is non-zero on consecutive intervals k only.

The nodal values t_i must make a monotone increasing sequence $(t_i \le t_{i+1})$. There may be real or integral values; they associate the variable u to the control points P_i. If the nodal values are equally distanced, the vector that they form is uniform, and the B-spline functions defined as such are uniform.

The uniform vector of the nodal values for an open curve is defined as folllows:

$t_i = 0$ for $i < k$

$t_i = i - k + 1$ for $k \le i \le n$

$t_i = n - k + 2$ for $i > n$

with

$0 \le i \le n + k$

In this case, the parametrical variable range is:

$$0 \le u \le n - k + 2$$

First we consider the expressions of the uniform B-spline functions of zero and 1 degree, in order to deduct the ones of second and third degree. We analyze the given curve by means of 6 control points. $(n = 5)$.

(1) Zero degree functions $(k = 1)$

$0 \le u \le 6$

$t_0 = 0$, $t_1 = 1$, $t_2 = 2$, $t_3 = 3$, $t_4 = 4$, $t_5 = 5$, $t_6 = 6$

$N_{0,1}(u) = 1$ for $0 \le u < 1$

=0 otherwise

$N_{1,1}(u) = 1$ for $1 \le u < 2$

=0 otherwise

....

$N_{5,1}(u) = 1$ for $5 \le u < 6$

=0 otherwise

We apply these functions to a random set of 6 control points and we have:

$p(u) = P_0$ for $0 \le u < 1$

$p(u) = P_1$ for $1 \le u < 2$

....

$p(u) = P_5$ for $5 \le u < 6$

The zero degree curve can be interpreted as being made of six segments of disjunct curve of zero length, each segment being concentrated at a control point.

(2) $(k = 2)$

$0 \le i \le 7$ $0 \le u \le 5$

$t_0 = 0$, $t_1 = 0$, $t_2 = 1$, $t_3 = 2$, $t_4 = 3$, $t_5 = 4$, $t_6 = 5$, $t_7 = 5$

$$N_{i,2}(u) = \frac{u - t_1}{t_{i+1} - t_i} * N_{i,1}(u) + \frac{t_i + 2 - u}{t_{i+2} - t_{i+1}} * N_{i+1,1}(u)$$

We must specify the expressions of the first degree functions for the new vector of the nodal values.

$N_{0,1}(u) = 1$ for $u = 0$

=0 otherwise

$N_{1,1}(u) = 1$ for $0 \le u < 1$

=0 otherwise

$N_{2,1}(u) = 1$ for $1 \le u < 2$

=0 otherwise

$N_{3,1}(u) = 1$ for $2 \le u < 3$

=0 otherwise

$N_{4,1}(u) = 1$ for $3 \le u < 4$

=0 otherwise

$N_{5,1}(u) = 1$ for $4 \le u < 5$

=0 otherwise

When we replace them in the expression of the second degree functions written above, we obtain:

$$N_{0,2}(u) = (1-u) * N_{1,1}(u)$$

$= 1-u$
for $0 \le u < 1$

= 0
otherwise

$$N_{1,2}(u) = u * N_{1,1}(u) + (2-u) - N_{2,1}(u)$$

$= u$
for $0 \le u < 1$

$= 2-u$
otherwise $1 \prec u < 2$

= 0
for

$$N_{2,2}(u) = (u-1) * N_{2,1}(u) + (3-u) * N_{3,1}(u)$$

$= u-1$
for $1 \le u < 2$

$= 3-u$
for $2 \le u < 3$

= 0
otherwise

$$N_{3,2}(u) = (u-2) * N_{3,1}(u) + (4-u) * N_{4,1}(u)$$

$= u-2$
for $2 \le u < 3$

$= 4-u$
for $3 \le u < 4$

=0
otherwise

$$N_{4,2}(u) = (u-3) * N_{4,1}(u) + (5-u) * N_{5,1}(u)$$

$= u-3$
for $3 \le u < 4$

$= 5-u$
for $4 \le u < 5$

= 0
otherwise

$$N_{5,2}(u) = (u-4) * N_{5,1}(u)$$

$= u-4$
for $4 \le u < 5$

= 0
otherwise

We apply a set of 6 control points and obtain the equation of the first degree B-spline curve determined by the respective points:

$$p(u) = P_0 * N_{0,2}(u) + P_1 * N_{1,2}(u) + ... + P_5 * N_{5,2}(u)$$

We can observe that on each interval $i \le u < i+1$, $0 \le i < 5$ only two mixture functions are non-zero, $N_{i,2}$ and $N_{i+1,2}$, so that the equation of the curve may be rewritten on portions:

$$p(u) = (1-u)P_0 + uP_1$$
for $0 \le u < 1$

$$p(u) = (2-u)P_1 + (u-1)P_2$$
for $1 \le u < 2$

$$p(u) = (3-u)P_2 + (u-2)P_3$$
for $2 \le u < 3$

....

The obtained curve is a sequence of segments of a line, that matches together the control points, with zero order continuity (C_0) at the connecting points.

(3) The second degree B-spline functions ($k = 3$)

$0 \le i \le 8$, $0 \le u \le 4$

Analogously:
- The vector of the nodal values is calculated:

$t_0 = 0$, $t_1 = 0$, $t_2 = 0$, $t_3 = 1$, $t_4 = 2$, $t_5 = 3$, $t_6 = 4$, $t_7 = 4$, $t_8 = 4$

- For obtaining the expressions of the functions $N_{i,3}(u)$, the expressions of the functions $N_{i,1}(u)$ must be determined first, then those of the function $N_{i,2}(u)$, this vector being different than the previous one:

$N_{0,1}(u) = 1$ for $u = 0$
= 0 otherwise

$N_{1,1}(u) = 1$ for $u = 0$
= 0 otherwise

$N_{2,1}(u) = 1$ for $0 \le u < 1$
= 0 otherwise

$N_{3,1}(u) = 1$ for $1 \le u < 2$
= 0 otherwise

$N_{4,1}(u) = 1$ for $2 \le u < 3$
= 0 otherwise

$N_{5,1}(u) = 1$ for $3 \le u < 4$
= 0 otherwise

$N_{0,2}(u) = 0$

$N_{1,2}(u) = (1-u)\ N_{2,1}(u)$

$N_{2,2}(u) = uN_{2,1}(u) + (2-u)\ N_{3,1}(u)$

$N_{3,2}(u) = (u-1)\ N_{3,1}(u) + (3-u)\ N_{4,1}(u)$

$N_{4,2}(u) = (u-2)\ N_{4,1}(u) + (4-u)\ N_{5,1}(u)$

$N_{5,2}(u) = (u-3)\ N_{5,1}(u)$

$N_{0,3}(u) = (1-u)^2\ N_{2,1}(u)$

= $(1-u)^2$ for $0 \le u < 1$
= 0 otherwise

$N_{1,3}(u) = 0{,}5\ u\ (4-3u)\ N_{2,1}(u) + 0{,}5\ (2-u)^2\ N_{3,1}(u)$

= $0{,}5\ u\ (4-3u)$ for $0 \le u < 1$
= 0,5 $(2-u)^2$ for $1 \le u < 2$
= 0 otherwise

$N_{2,3}(u) = 0{,}5\ u^2\ N_{2,1}(u) + 0{,}5\ (-2u^2 + 6u - 3)\ N_{3,1}(u) + 0{,}5\ (3-u)^2\ N_{4,1}$

= 0,5 u^2 for $0 \le u < 1$

= 0,5 ($-2u^2$ +6u -3)
for $1 \le u < 2$

= 0,5 (3-$u)^2$
otherwise $2 \le u < 3$

= 0
otherwise

$N_{3,3}(u) = 0,5\ (u-1)^2\ N_{3,1}(u) + 0,5\ (-2\ u^2 + 10\ u$ -11) $N_{4,1}(u) + 0,5\ (4-u)^2$ $N_{5,1}(u)$

= 0,5 $(u-1)^2$
for $1 \le u < 2$

= 0,5 $(-2\ u^2 + 10\ u$ -1)
for $2 \le u < 3$

= 0,5 $(4-u)^2$
for $3 \le u < 4$

= 0
otherwise

$N_{4,3}(u) =$ 0,5 $(u-2)^2\ N_{4,1}(u)$ +0,5 $(-3\ u^2$ +20u -32) $N_{5,1}(u)$

= 0,5 $(u-2)^2$
for $2 \le u < 3$

= 0,5 $(-3\ u^2$ +20u -32)
for $3 \le u \le 4$

= 0
otherwise

$N_{5,3}(u) = (u-3)^2\ N_{5,1}(u)$

= $(u-3)^2$
for $3 \le u \le 4$

= 0
otherwise

One can notice that only 3 mixture functions are non-zero on each interval $i \le u < i+1$, $0 \le i < 4$. As in the previous case, the second degree B-spline equation can be written on intervals.

The polynomial expression that defines the curve on the interval $i \to i+1$ is noted $p_{i+1}(u)$. As such, the following is obtained:

$p_1(u) = (1-u)^2 P_0$ +0,5u $(4-3\ u)P_1$ +0,5$u^2 P_2$

$p_2(u) =$ 0,5 $(2-u)^2 P_1$ +0,5 $(-2\ u^2 + 6\ u - 3)P_2$ +0,5$(u-1)^2 P_3$

$p_3(u) =$ 0,5 $(3-u)^2 P_2$ +0,5 $(-2\ u^2 + 10\ u - 11)P_3$ +0,5$(u-2)^2 P_4$

$p_4(u) =$ 0,5 $(4-u)^2 P_3$ +0,5 $(-3\ u^2 + 20\ u - 32)P_4$ + $(u-3)^2 P_5$

The obtained curve is made out of 4 curve segments connected with C^1 continuity. The curve gous only through the points P_0, P_5, is tangent at the segments $P_1 - P_0$, $P_5 - P_4$ and also is tangent at each side of the characterisctic polygon.

Each curve segment is influenced only by three consequent control points, or a control point can influence the form of three curve segments only. The conclusion may also be drawn from the results obtained for $k = 1$ and $k = 2$. Each segment of a B-spline curve is influenced only by k consequent control points, or each control point influences k curve segments only.

The mixture functions attached to the ‚interior' control points are independent from the number of control points. For example, for $k = 3$, $n = 5$, $N_{2,3}$ and $N_{3,3}$ having the same form, being declared with an interval on the u axis. A mathematical expression of the mixture functions is necessary, independent of the number of the control points. Thus, we determine the expression of the functions $N_{i,3}(u)$, $N_{i+2,3}(u)$ for a random interval. In order that the values have the same expression, we consider. This results in:

$N_{i,1}(u) = 1$ for $i-2 \le u < i-1$

$= 0$ otherwise

$N_{i+1,1}(u) = 1$ for $i-1 \le u < 1$

$= 0$ otherwise

$N_{i+2,1}(u) = 1$ for $i \le u < i+1$

$= 0$ otherwise

$N_{i+3,1}(u) = 1$ for $i+1 \le u < i+2$

$= 0$ otherwise

$N_{i+4,1}(u) = 1$ for $i+2 \le u < i+3$

$= 0$ otherwise

$N_{i,2}(u) = (u-i+2)\ N_{i,1}(u) + (i-u)\ N_{i+1,1}(u)$

$N_{i+1,2}(u) = (u-i+1)\ N_{i+1,1}(u) + (i+1-u)\ N_{i+2,1}(u)$

$N_{i+3,2}(u) = (u-i)\ N_{i+2,1}(u) + (i+2-u)\ N_{i+3,1}(u)$

$N_{i+3,2}(u) = (u-i-1)\ N_{i+3,1}(u) + (i+3-u)\ N_{i+4,1}(u)$

$N_{i,3}(u) =$ 0,5 $[(u-i+2)^2\ N_{i,1}(u) + (u-i+2)(i-u)\ N_{i+1,1}(u) + (i+1-u)(u-i+1)\ N_{i+1,1}(u) + (i+1-u)^2\ N_{i+2,1}(u)]$

$N_{i+1,3}(u) =$ 0,5 $[(u-i+1)^2\ N_{i+1,1}(u) + (u-i+1)(i+1-u)\ N_{i+2,1}(u) + (i+2-u)(u-i)\ N_{i+2,1}(u) + (i+2-u)^2\ N_{i+3,1}(u)]$

$N_{i+2,3}(u) =$ 0,5 $[(u-i)^2\ N_{i+2,1}(u) + (u-i)(i+2-u)\ N_{i+3,1}(u) + (i+3-u)(u-i-1)\ N_{i+3,1}(u) + (i+3-u)^2\ N_{i+4,1}(u)]$

We observe that $N_{i,3}(u) = N_{i+1,3}(u+1) = N_{i+2,3}(u+2)$. The three functions have the same form, but on consequent intervals. Therefore, it is better to give up the variable i in expressing them, that is, to get to a normalized form, in which u takes values within the range [0,1) on an interval $[i, i+1)$. In the functional expressions, we replace the variable u with $u^* = u - i$, make the substitution $u = u^* + i$, but in order to make the notation easier we will note $u^* + i$ with $u + i$. We obtain:

$N_{i,3}(u+i) =$ 0,5 $(u+2)^2$ for $-2 \le u < -1$

$=$ 0,5 $[-u(u+2) + (1-u)(u+1)]$ for $-1 \le u < 0$

= 0,5 $(1-u)^2$

for $0 \le u < 1$

= 0

otherwise

$N_{i+1,3}(u+i)$ = 0,5 $(u+1)^2$

for $-1 \le u < 0$

= 0,5 $[(u+1)(1-u)+(2-u)u]$

for $0 \le u < 1$

= 0,5 $(2-u)^2$

for $1 \le u < 2$

= 0

otherwise

$N_{i+2,3}(u+i)$ = 0,5 u^2

for $0u < 1$

= 0,5 $[u(2-u)+(3-u)(u-1)]$

for $1 \le u < 2$

= 0,5 $(3-u)^2$

for $2 \le u < 3$

= 0

otherwise

The B-splines defined like this are called periodic B-Splines. They have the same form, but are staggered on the u axis, each being non-zero on only three consequent intervals.

We will represent the equation of the segment of the second degree B-spline curve, for an interval $i \le u < i+1$, using the normalized forms, $0 \le u < 1$:

$$p_i(u) = 0{,}5\,[(1-u)^2 P_i + (-2u^2+2u+1)P_{i+1} + u^2 P_{i+2})]$$

The equations of the 4 segments of the second degree B-spline curve by means of which the B-spline determined by 6 control points ($n=5$) is approximated, can be written in a matrix form:

$$p_{i+1}(u) = [F_{1,3}(u) \quad F_{2,3}(u) \quad F_{3,3}(u)]\,[P_i \; P_{i+1} \; P_{i+2}]^T \qquad 0 \le i \le 3$$

Or

$$p_i(u) = [F_{1,3}(u) \; F_{2,3}(u) \; F_{3,3}(u)]\,[P_{i-1} \; P_i \; P_{i+1}]^T \; 1 \le i \le 4$$

Where:

$F_{1,3}(u)$ = 0,5 (u^2-2u+1)

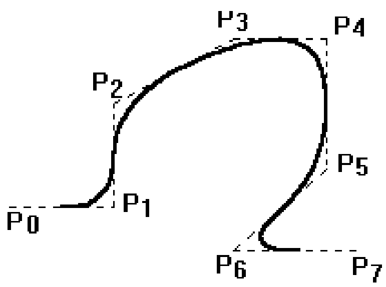
$F_{2,3}(u)$ = 0,5 $(-2u^2+2u+1)$

$F_{3,3}(u)$ = 0,5 u^2

are the mixture functions for a uniform, periodic, second degree B-spline curve segment.

The following matrix equation represents a uniform, periodic, second degree B-spline curve, resulted from a $n+1$ control points.

$$p_i(u) = 0{,}5\,[u^2 \quad u \quad 1] \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \end{bmatrix} \begin{matrix} 1 \le i \le n-1 \\ 0 \le u < 1 \end{matrix}$$



The similar form for the uniform, periodic, third degree B-spline curves is:

$$p_i(u) = 1/6[u^3 \quad u^2 \quad u \quad 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix} \begin{matrix} 1 \le i \le n-2 \\ 0 \le u < 1 \end{matrix}$$

We note:

$$U_3 = [u^2 \, u \quad 1]$$

$$U_4 = [u^3 \, u^2 \, u \quad 1]$$

$$M_3 = 1/2 \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_4 = 1/6 \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$ - the basic B-spline matrix

Using these notations, we will rewrite the matrix equations of the B-spline curves

- for second degree open curves:

$$p_i(u) = U_3 M_3 \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \end{bmatrix}$$

$1 \le i \le n-1$

$0 \le u < 1$

- For third degree open curves:

$$p_i(u) = U_4 M_4 \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix}$$

$1 \le i \le n-2$

$0 \le u < 1$

- For second degree closed curves:

$$p_i(u) = U_3 M_3 \begin{bmatrix} p_{(i-1) \bmod (n+1)} \\ p_{i \bmod (n+1)} \\ p_{(i+1) \bmod (n+1)} \end{bmatrix}$$

$1 \le i \le n+1$

$0 \le u < 1$

- For third degree closed curves:

$$p_i(u) = U_4 M_4 \begin{bmatrix} p_{(i-1) \bmod (n+1)} \\ p_{i \bmod (n+1)} \\ p_{(i+1) \bmod (n+1)} \\ p_{(i+2) \bmod (n+1)} \end{bmatrix}$$

$1 \le i \le n+2$

$0 \le u < 1$

Where $a \bmod b$ is the result of dividing a to.

Properties of the B-spline Curves

1. Multiple control points

An m degree B-spline curve goes always through a control point of m multiplicity. By introducing a control point in the vector of the control points on more consequent positions, one can force the B-spline curve going through the respective point.

2. Collinear control points

If $m+1$ consequent control points are located on a line, then the m degree B-spline curve is located partially on the respective line. If the points P_{i-1}, P_i, P_{i+1} are collinear, the segment p_i of the second degree B-spline curve will partially intermingle with the segment $P_{i-1} - P_i$.

3. Closed curves

In order to make a m degree B-spline closed curve, it is sufficient that the first m control points are identical with the m endpoints.

4. The convex hull property

A B-spline curve is completely included in the convex polygon formed by matching together the control points.

5. Affine invariance

In order to transform a B-spline curve, it is sufficient to apply the transformation of the control points and then to regenerate the curve.

Properties of the second degree B-spline curves

As they are more predictable than the cubic, the second degree B-spline curves are congruous for the representation of the high precision text characters. The characters of the True Type set utilized in the Windows system are defined utilizing second degree B-spline curves. Any second degree B-spline curve has withal the following properties:

- Goes through the point located in the middle of the distance between two consequent control points؛
- at this point, the tangent of the curve intermingles with the segment that matches the two control points .
- The curve is located in the triangle defined by a control point, and the middle of the segments $P_{i-1} - P_i, P_i - P_{i+1}$

2.5 Surfaces

The surface modelling is one of the most important problems in the computerized design and production systems. The models must offer the design flexibility, this being a creative type of activity, must be able to achieve simple implementations of the calculations of the surface properties and must allow the representation of a high variety of forms.

A surface can be mathematically determined in three ways:

(1) by means of an implicit equation as:

$F(x, y, z) = 0$

(2) By means of an explicit equation, which expresses the variance of one of the three variants based on the other two:

$x = f_x(y, z)$ or $y = f_y(x, z)$ or $z = f_z(x, y)$

(4) By means of parametric equations

$x = f_x(u, v)$

$y = f_y(u, v)$ $u\min \leq u \leq u\max$

$z = f_z(u, v)$ $v\min \leq v \leq v\max$

As in the case of the curves, the parametric equations offer many advantages as compared to the other design methods, among which we can mention here:

- The representation is independent of any coordinate system;
- One can represent surfaces defined by means of functions with multiple values;
- The 3D transformations expressed in homogeneouscoordinates can be applied directly on parametric equations;
- The surfaces defined parametrically are inherently limited, through the variation range of the parametric variables; any portion of the surface can be defined by choosing the range for each variable;
- The parametric equations offer a higher degree of freedom for the control of a surface.

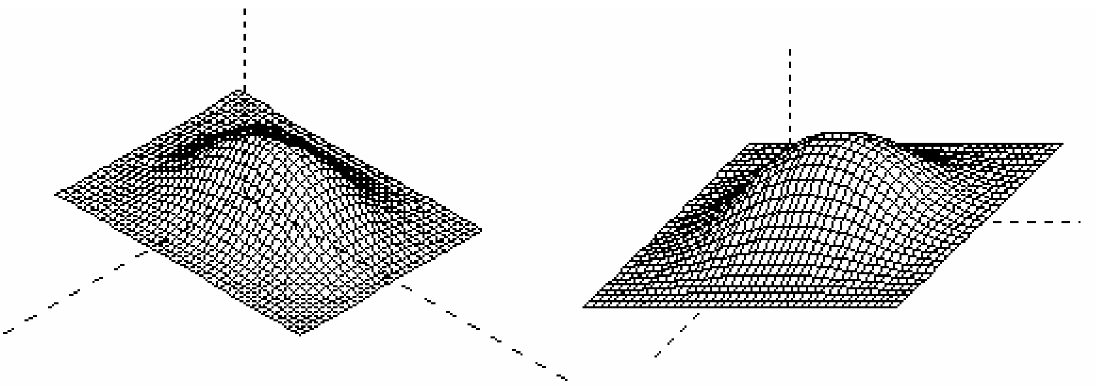
The parametric equations can be used to show a high variety of surfaces, such as the ones obtained by means byspecial scanning and the Freeform surfaces.

2.6 B-spline Surfaces

The B-spline surfaces are approximation surfaces defined only by points. They are represented by the following parametric equation:

$$P(u,w)=\sum_{i=0}^{m}\sum_{j=0}^{n}p_{ij} * N_{i,k}(u)*N_{j,l}(w)$$

Where p_{ij} are the control points that define the surface, and $N_{j,l}(w)$ are B-spline functions of degree $(k-1)$ and $(l-1)$ respectively.



A B

The image above, in isometric position (a) and Cavalier (b), shows the biquadratic B-spline surface $(l=k=3)$ defined by the following matrix of the control points $(m=5, n=4)$:

$$P=\begin{bmatrix} (-100,0,100) & (-100,0,50) & (-100,0,0) & (-100,0,-50) & (-100,0,-100) \\ (-50,0,100) & (-50,0,50) & (-50,0,0) & (-50,0,-50) & (-50,0,-100) \\ (0,0,100) & (0,0,50) & (0,70,0) & (0,0,-50) & (0,0,-100) \\ (50,0,100) & (50,0,50) & (50,70,0) & (50,0,-50) & (50,0,-100) \\ (100,0,100) & (100,0,50) & (100,0,0) & (100,0,-50) & (100,0,-100) \\ (150,0,100) & (150,0,50) & (150,0,0) & (150,0,-50) & (150,0,-100) \end{bmatrix}$$

A B-spline curve is obtained by means of the juxtaposition of more curve segments. The equations that define the segments of the second and third degree periodic B-spline curve can be rewritten as follows:

$$p_s(u) = U_k * M_k * P_k$$
$$1 \le s \le n+2-k$$

Where

$$P_k = \{P_i \mid s-1 \le i \le s+k-2\}$$

And n is the number of the control points.

For $k=3$ we obtain the matrix expression of the segment of the first order B-spline curve (second degree), and for $k=4$ we obtain the expression of the segment of the second order B-spline curve (third degree). A B-spline surface (open, periodic) is obtained by the juxtaposition of more surface segments, defined as follows:

$$p_{st}(u,w) = U_k * M_k * P_{kl} * M_l^T * W_l^T$$

$$1 \le s \le m+2-k$$

$$1 \le t \le n+2+l \quad 0 \le u,w \le 1$$

$$p_{kl} = \{p_{ij} \mid s-1 \le i \le s+k-2, t-1 \le j \le t+l-2\}$$

By introducing $k=4$ and $l=4$ in the above equation, we obtain the equation of the segment of the B-spline bicubic surface. We can observe that each surface segment is determined by 4×4 control points.

In practice, for the realization of the division operations of the B-spline bicubic patches, a conversion of the patches from the B-spline representation to the Bézier representation is being used.

If

$$S = U * M_S * P_S * M_S{}^T * W^T$$

the equation of the B-spline bicubic patch and

$$B = U * M_B * P_B * M_B{}^T * W^T$$

the equation of the Bézier bicubic patch.

The conversion of the B-spline patch into Bézier patch assumes the determination of the matrix of the control points P_B, which determines the patch in the Bézier representation. For this, the following condition is necessary:

$$U * M_S * P_S * M_S{}^T * W^T = U * M_B * P_B * M_B{}^T * W^T$$

or

$$M_S * P_S * M_S{}^T = M_B * P_B * M_B{}^T$$

We obtain:

$$P_B = [M_B^{-1} * M_S]* P_S * [M_B^{-1} * M_S]^T$$

or

$$P_B = A * P_S * A^T$$

where

$$A = 1/6 \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix}$$

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المستخلص

البحث يسلط الضوء على الافكار الاساسية ذات الطبيعة المشابهة لمنحنيات الشرائح والشرائح من نوع (B) والسطوح لمعرفة استخداماتها في تطبيقات فهم الاشارات المبهمة. ومن خلال استخدام سطوح المنحنيات، يمكن للمرء ان ينشئ خوارزميات كشف الهوية للمصادر المبهمة، عن طريق تخصيص استعمالها سويةً مع تقنيات حقل (kenned)، اضافة الى ذلك، تستخدم هذه التقنيات على نطاق واسع في الرسومات المحوسبة للنمذجة والتصميم، كما تمتلك العديد من الخصائص الهندسية والقدرات الحسابية.