The Cyclic decomposition and the Artin characters table of the group ($Q_{2m} \times C_p$) when $m=2^h$, $h \in Z^+$ and p is prime number

Ass. Lecturer Rajaa Hassan Abass

Email: rajaah.alabidy@uokufa.edu.iq

University of Kufa, College of Education for Girls, Department of Mathematics

Abstract

The main purpose of this paper, is determination of the cyclic decomposition of the abelian factor group AC(G) = $\overline{R}(G)/T(G)$ where G = $Q_{2m} \times C_p$ when $m=2^h$, $h \in Z^+$ and p is prime number (the group of all Z-valued characters of G over the group of induced unit characters from all cyclic subgroups of G).

We have found that the cyclic decomposition AC($Q_{2m} \times C_p$) depends on the elementary divisor of m as follows.

if $m = 2^h$, h any positive integer and p is prime number, then:

$$AC(Q_{2m} \times C_p) = \bigoplus_{i=1}^{2(h+1)} C_2$$

We have also found the general form of Artin's characters table of Ar($Q_{2m} \times C_p$) when $m=2^h$, $h \in Z^+$ and p is prime number.

Introduction:

The problem of determining the cyclic decomposition of AC(G) seem to be untouched. We use the concepts of invariant matrix in linear algebra to find the cyclic decomposition of AC(G), G is considered to be the group $Q_{2^{h+1}} \times C_p$.

In 1968 T.Y Lam [13] defined AC(G) and he studied AC(G),when G is a cyclic group.

In 2000 H.R .Yassin [4] studied the cyclic decomposition of AC(G) when G is an elementary abelian group . In 2006 A.S. Abid [2] found Ar(Cn) when Cn is the cyclic group of order n .

In this paper, we find the cyclic decomposition of the factor group AC($Q_{2^{h+1}} \times C_p$) and the Artin characters table where Q_{2m} is the Quaternion group of order 4m When $m=2^h$, $h \in Z^+$ and C_p is the Cyclic group of order p, p is prime number.

1.Some Basic Concepts:

In this section, we give basic concepts, notations and theorems about the group ($Q_{2m} \times C_p$), a rational valued characters, a rational valued characters table, the Artin characters and the Artin characters table.

The Group ($Q_{2m} \times C_p$)(1.1):

The direct product group ($Q_{2m} \times C_p$) where Q_{2m} is Quaternion group of order 4m with tow generators x and y is denoted by

$$Q_{2m} = \{x^k y^j : x^{2m} = y^4 = 1, yx^m y^{-1} = x^{-m}, 0 \leq k \leq 2m-1, j=0,1\}$$

and C_p is acyclic group of order p consisting of elements $\{I,z,z^2,\dots,z^{p-1}\}$ when p is prime number .the generalized the group $(Q_{2m} \times C_p)$ is denoted by

$(Q_{2m} \times C_p) = \{(q,c):q \in Q_{2m}, c \in C_p\}$ and

$|Q_{2m} \times C_p| = |Q_{2m}|.|C_p| = 4m.p = 4p.m$

Definition (1.2):[8]

A rational valued character θ of G is a character whose values are in Z, which is $\theta(g) \in Z$, for all $g \in G$

Corollary (1.3):[9]

The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$

form the basis for $\overline{R}(G)$, where χ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of cyclic subgroup of G .

Proposition (1.4):[7]

The number of all rational valued characters of a finite group G is equal to the number of all distinct Γ- classes on G.

Definition (1.5): [9]

The complete information about rational valued characters of a finite group G is displayed in a table called **rational valued characters table of G**. We refer to it by $\overset{*}{\equiv}(G)$ which is n×n matrix whose columns are Γ-classes and rows which are the values of all rational valued characters of G, where n is the number of Γ-classes.

Proposition (1.6):[10]

The general form of the rational valued characters table of the Quaternion group Q_{2m} when $m=2^h$, h is any positive integer and it is given by:

$$\overset{*}{\equiv}(Q_{2m}) = \overset{*}{\equiv}(Q_{2.2^h}) =$$

Γ-classes	[1]	$[x^{2^h}]$	$[x^{2^{h-1}}]$	$[x^{2^{h-2}}]$	·	$[x^2]$	[x]	[y]	[xy]
θ_1	2^h	-2^h	0	0	·	0	0	0	0
θ_2	2^{h-1}	2^{h-1}	-2^{h-1}	0	·	0	0	0	0
θ_3	2^{h-2}	2^{h-2}	2^{h-2}	-2^{h-2}	·	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮
θ_{l-2}	2	2	2	2	·	-2	0	0	0
θ_{l-1}	1	1	1	1	·	1	-1	-1	1
θ_l	1	1	1	1	·	1	1	-1	-1
θ_{l+1}	1	1	1	1	·	1	-1	1	-1
θ_{l+2}	1	1	1	1	·	1	1	1	1

Table (1)

Where l is the number of Γ-classes of C_m .

Theorem(1.7):[6]

The rational valued characters table of the group $(Q_{2m} \times C_p)$ is equal to the tensor product of the rational valued characters table of Q_{2m} and the rational valued characters table of C_p when p is prime number that is:

$$\overset{*}{\equiv}(Q_{2m} \times C_p) = \overset{*}{\equiv}(Q_{2m}) \otimes \overset{*}{\equiv}(C_p) \ .$$

Theorem(1.8): [5]

Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen as representative for m-conjugate classes of H contained in CL(g) in G, then :

1- $\varphi'(g) = \dfrac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$ if $H \cap CL(g) = \phi$.

Definition(1.9):[13]

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called **Artin's characters of G.**

In theorem (1.8) , if φ is the principal character , then $\varphi(h_i) = \varphi(1) = 1$, where $h_i \in H$

Proposition(1.10):[3]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.

Furthermore, Artin's characters are constant on each Γ-classes.

Definition(1.11): [2]

Artin's characters of finite group G can be displayed in a table **called Artin's characters table of G** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest rows contain the values of Artin's characters.

Theorem(1.12):[2]

The general form of Artin's character table of C_{p^s} when p is a prime number and s is an integer number is given by:

Γ-classes	[1]	p^{s-1}]	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	$[x^p]$	$[x]$
$\lvert CL_\alpha \rvert$	**1**	**1**	**1**	**1**	…	**1**	**1**
$\lvert C_{p^s}(CL_\alpha) \rvert$	P^s	p^s	P^s	p^s	…	p^s	P^s
φ'_1	p^s	**0**	**0**	**0**	…	**0**	**0**
φ'_2	P^{s-1}	P^{s-1}	**0**	**0**	…	**0**	**0**
φ'_3	P^{s-2}	P^{s-2}	P^{s-2}	**0**	…	**0**	**0**
⋮	⋮	⋮	⋮	⋮	.	⋮	⋮
φ'_s	**P**	**P**	**P**	**P**	…	**P**	**0**
φ'_{s+1}	**1**	**1**	**1**	**1**	…	**1**	**1**

Table (2)

Proposition (1.13): [12]

The Artin's characters table of the Quaternion group Q_{2m} when $m=2^h$, $h \in Z^+$ is given as follows

$Ar(Q_{2^{h+1}})=$

Γ- classes	Γ- classes of C_{2m}					
	[1]	$[x^{2^h}]$			[y]	[xy]
$\|CL_\alpha\|$	1	1	2 2 …	2	2^h	2^h
$\|C_{Q_{2^{h+1}}}(CL_\alpha)\|$	2^{h+2}	2^{h+2}	2^{h+1} 2^{h+1} …	2^{h+1}	4	4
Φ_1	$2Ar(C_{2^{h+1}})$				0	0
Φ_2					0	0
⋮					⋮	⋮
Φ_l					0	0
Φ_{l+1}	2^h	2^h	0 0 …	0	2	0
Φ_{l+2}	2^h	2^h	0 0 …	0	0	2

Table (3)

where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \leq j \leq l+2$ are the Artin characters of the Quaternion group Q_{2m} when $m=2^h$, $h \in Z^+$

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2.the Factor Group AC(G):

This section is devoted to the study of the factor group AC(G) of a group G .

Definition(2.1):[9]

Let T(G) be the subgroup of $\overline{R}$ (G) generated by Artin's characters .T (G) is normal subgroup of $\overline{R}$ (G) and denotes the factor abelian group $\overline{R}$ (G)/T(G) by AC(G) which is called **Artin cokernel of** G.

Definition(2.2):[8]

Let M be a matrix with entries in a principal domain R. **A k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

Definition(2.3):[8]

A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M.

This is denoted by D_k (M)

Lemma(2.4):[8]

Let M, P and W be matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then D_k (P M W)= D_k (M) module the group of unites of R.

Theorem(2.5):[8]

Let M be an $n \times n$ matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

1- P and W are invertible.
2- P M W = D.
3- D is diagonal matrix.
4- if we denote D_{ii} by d_i then there exists a natural number m ; $0 \leq m \leq n$ such that $j > m$ implies $d_j = 0$ and $j \leq m$ implies $d_j \neq 0$ and $1 \leq j \leq m$ implies $d_j | d_{j+1}$.

Definition(2.6):[8]

Let M be matrix with entries in a principal domain R, be equivalent to a matrix D = diag $\{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$ such that $d_j | d_{j+1}$ for $1 \leq j < m$

We call D **the invariant factor matrix of** M and $d_1, d_2, \dots, d_m$ the invariant factors of M

Theorem(2.7):[8]

Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal $\langle d_1 \rangle, \langle d_2 \rangle, \dots, \langle d_m \rangle$, $d_j | d_{j+1}$ for j = 1, 2, … , K-1.

3.The Matrix M(G) :

This section is devoted to the study of the matrix M(G), $M(Q_{2m})$, $P(Q_{2m})$ and $W(Q_{2m})$.

Proposition(3.1):[9]

AC(G) is a finitely generated Z- module .Let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l. There exists an invertible matrix M(G) with entries in rational number such

That: $\equiv^*(G)=M^{-1}(G).Ar(G)$ and this implies $M(G)=Ar(G).(\equiv^*(G))^{-1}$

Theorem(3.2):[4]

$$AC(G) = \bigoplus_{i=1}^{l} C_{d_i} \quad \text{where } d_i = \pm D_i(G)/D_{i-1}(G)$$

where l is the number of all distinct Γ-classes.

Corollary(3.3):[9]

$$|AC(G)| = |\det(M(G))|.$$

Proposition(3.4):[11]

If p is prime number and s is positive integer, then:

$$M(C_{p^s}) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

which is of the order (s+1)×(s+1)

Proposition(3.5):[11]

The general form of the matrices $P(C_{p^s})$ and $W(C_{p^s})$ is:

$$P(C_{p^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

which is (s+1)×(s+1) square matrix.

$W(C_{p^s}) = I_{s+1}$, where I_{s+1} is (s+1)×(s+1) identity matrix

and $D(C_{p^s}) = \text{diag}\,\underbrace{\{1,1,.......,1\}}_{s+1}$.

Remarks(3.6): [1]

if $m=2^h$,h is any positive integer ,then we can write $M(C_m)$ as the following :

$$M(C_m) = \begin{bmatrix} & & & & 1 & 1 \\ & & & & 1 & 1 \\ & & R_1(C_m) & & \vdots & \vdots \\ & & & & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

which is (h+1)×(h+1) square matrix ,$R_1(C_m)$ is the matrix obtained by omitting the last two rows $\{0,0,\cdots,1,1\}$ and $\{0,0,\cdots,0,0,1\}$ and the last two columns $\{1,1,\cdots,1,0\}$ and $\{1,1,\cdots,1,1\}$ from the matrix $M(C_{2^h})$ in the Proposition (3.6).

Proposition(3.7):[12]

If $m=2^h$,h any positive integers ,then the matrix $M(Q_{2m})$ of the quaternion group Q_{2m} is :

$$M(Q_{2m}) = \left[\begin{array}{cccccc|cccc} & & & & & & 1 & 1 & 1 & 1 \\ & 2R_1(C_{2m}) & & & & & 1 & 1 & 1 & 1 \\ & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & \cdots & \cdots & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

which is (h+4)×(h+4) square matrix ,$R(C_{2m})$ is similar to the matrix in the remarks (3.8).

Proposition(3.8):[12]

If $m=2^h$,h any positive integer then the matrices $P(Q_{2m})$ and $W(Q_{2m})$ are taking the forms:

$$P(Q_{2m})=\begin{bmatrix} & & & & & & 0 & 0 \\ & & & & & & 0 & 0 \\ & & P(C_{2m}) & & & & \vdots & \vdots \\ & & & & & & \vdots & \vdots \\ & & & & & & 0 & 0 \\ & & & & & & -1 & 1 \\ & & & & & & 0 & -1 \\ 0 & 0 & \cdots & & \cdots & 0 & 1 & -1 \\ 0 & 0 & \cdots & & \cdots & 0 & 0 & 1 \end{bmatrix}$$

$$and\quad W(Q_{2m})=\begin{bmatrix} & & & & & & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 \\ & & I_{h+1} & & & & \vdots & \vdots & \vdots \\ & & & & & & \vdots & \vdots & \vdots \\ & & & & & & 0 & 0 & 0 \\ 0 & 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & \cdots & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

where I_{h+1} is the identity matrix .They are (h+4)×(h+4) square matrix .

Example (3.9):

To find $P(Q_{32})$ and $W(Q_{32})$, by the proposition (3.8).

$$P(Q_{32})=P(Q_{2^5})=\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

which is 8×8 square matrix

and

$$W(Q_{32})=W(Q_{2^5})=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

which is 8×8 quare matrix

4.The Main Results

In this section we give the general from of the Artin's characters table of the group $(Q_{2^{h+1}}\times C_p)$ and the cyclic decomposition of the factor group $AC(Q_{2m}\times C_p)$ when $m=2^h$, $h\in Z^+$ and p is prime number. .

Proposition(4.1):

The general from of the Artin's characters table of the group $(Q_{2^{h+1}}\times C_p)$ when $m=2^h, h\in Z^+$ and p is prime number is give as follows:

$Ar(Q_2^{h+1}\times C_p)=$

	Γ- classes of $(Q_2^{h+1})\times\{I\}$						Γ- classes of $(Q_2^{h+1})\times\{z\}$					
Γ- classes	**[1,I]**	$[x^{2^h},I]$	**…**	**[x,I]**	**[y,I]**	**[xy,I]**	**[1,z]**	$[x^{2^h},z]$	**…**	**[x,z]**	$[y,z]$	$[xy,z]$
$\lvert CL_\alpha \rvert$	**1**	**1**	**…**	**2**	2^h	2^h	**1**	**1**	**…**	**2**	2^h	2^h
$\lvert C_{Q_2^{h+1}\times C_p}(CL_\alpha) \rvert$	$p2^{h+2}$	$p2^{h+2}$	**…**	$p2^{h+1}$	**4p**	**4p**	$p2^{h+2}$	$p2^{h+2}$	**…**	$p2^{h+1}$	**4p**	**4p**
$\Phi_{(1,1)}$	$pAr(Q_2^{h+1})$						**0**					
$\Phi_{(2,1)}$												
⋮												
$\Phi_{(l,1)}$												
$\Phi_{(l+1,1)}$												
$\Phi_{(l+2,1)}$												
$\Phi_{(1,2)}$	$Ar(Q_2^{h+1})$						$Ar(Q_2^{h+1})$					
$\Phi_{(2,2)}$												
⋮												
$\Phi_{(l,2)}$												
$\Phi_{(l+1,2)}$												
$\Phi_{(l+2,2)}$												

Table (4)

Proof :

Let $g \in (Q_{2m}\times C_p)$;g=(q,I) or g=(q,z)or $g=(q,z^2)$…

$g=(q,z^{p-1})$,$q \in Q_{2m}$ when $m=2^h$,$h\in Z^+$,I,z,z^2,…$z^{p-1} \in C_p$

Case (I):

If H is a cyclic subgroup of $Q_{2m}\times \{I\}$,then:

2. $H=\langle(y,I)\rangle$

1.$H=\langle(x,I)\rangle$ 3. $H=\langle(xy,I)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (1.8)

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i\in H\cap CL(g)\\ 0 & if \quad H\cap CL(g)=\phi\end{cases}$$

1. IF $H=\langle(x,I)\rangle$

(i) if $g=(1,I), g\in H$

$$\Phi_{(j,1)}((1,I))=\frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{p.2^{h+2}}{|C_H(I,1)|}.1=\frac{p.|C_{Q_{2^{h+1}}}(1)|}{|C_{\langle x\rangle}(1)|}.\varphi(1)=p.\Phi_j(1),$$

since $H\cap CL(1,I)=\{(1,I)\}$

(ii) if $g=((x^{2^h},I), g\in H$

$$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{p.2^{h+2}}{|C_H(g)|}.1=\frac{p.|C_{Q_{2^{h+1}}}(x^{2^h})|}{|C_{\langle x\rangle}(x^{2^h})|}.\varphi(g)=p.\Phi_j(x^{2^h})$$

since $H\cap CL(g)=\{g\}$,$\varphi(g)=1$

(iii) if $g\neq(x^{2^h},I), g\in H$

$$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=$$
$$\frac{p.2^{h+1}}{|C_H(g)|}.(1+1)=$$

$$\frac{p.2^{h+1}}{|C_H(g)|}.(1+1)=\frac{p.|C_{Q_{2^{h+1}}}(q)|}{|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=p.\Phi_j(q)$$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$,g=(q,I),$q\in Q_{2^{h+1}}$ and $q\neq x^{2^h}$

(iv) if $g\notin H$

$\Phi_{(j,1)}(g)=p.0=p.\Phi_j(q),$ Since $H\cap CL(g)=\phi$

2. IF $H=\langle(y,I)\rangle=\{(1,I),(y,I),(y^2,I),(y^3,I)\}$

(i) If g=(1,I) $H\cap CL(1,I)=\{(1,I)\}$

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{p.4.2^h}{4}.1=p.2^h=p.\Phi_{l+1}(1)$$

(ii) If $g=(x^{2^h},I)=(y^2,I)$ and $g\in H$

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{p.4.2^h}{4}.1=p.2^h=p.\Phi_{l+1}(x^{2^h})$$

Since $H\cap CL(g)=\{g\}$,$\varphi(g)=1$

(iii)If $g\neq(x^{2^h},I)$ and $g\in H$,i.e.{g=(y,I) or $g=(y^3,I)$}

Otherwise

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{p.4}{4}.(1+1)=p.2=p.\Phi_{l+1}(y)$$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise

$$\Phi_{(l+1,1)}(g) = 0 \quad \text{since } H\cap CL(g)=\emptyset$$

3-IF H=$\langle(xy, I)\rangle$

$=\{(1,I),(xy,I),((xy)^2,I)=(y^2,I),((xy)^3,I)=(xy^3,I)\}$

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{p.4.2^h}{4}.1 = p.2^h = p.\Phi_{l+2}(1)$$

(ii) If g= (x^{2^h}, I)=$((xy)^2,I)=(y^2,I)$ and g∈ H

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{p.4.2^h}{4}.1 = p.2^h = p.\Phi_{l+2}(x^{2^h})$$

Since H∩CL(g)={g} , φ (g)=1

(iii)If g≠(x^{2^h}, I) and g∈ H,i.e.{g=(xy,I)or g=$((xy)^3,I)$}

$$\Phi_{(l+2,1)}(g) = \frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$= \frac{p.4}{4}.(1+1) = p.2 = p.\Phi_{l+2}(xy)$$

since H∩CL(g)=$\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

$$\Phi_{(l+2,1)}(g) = 0 \quad \text{since } H\cap CL(g)=\phi$$

Case (II):

If H is a cyclic subgroup of ($Q_2^{h+1}\times$ {z}),then

1.H=$\langle(x,z)\rangle = \langle(x,z^2)\rangle$ =

$\langle(x,z^3)\rangle = ... = \langle(x,z^{p-1})\rangle$

2.H=$\langle(y,z)\rangle = \langle(y,z^2)\rangle$ =

$\langle(y,z^3)\rangle = ... = \langle(y,z^{p-1})\rangle$

3.H=$\langle(xy,z)\rangle = \langle(xy,z^2)\rangle$ =

$\langle(xy,z^3)\rangle = ... = \langle(xy,z^{p-1})\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_2^{h+1} $1\le j\le l+2$, then by using theorem (1.8)

$$\Phi_j(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i\in H\cap CL(g) \\ 0 & if \quad H\cap CL(g)=\phi \end{cases}$$

1.IFH=$\langle(x,z)\rangle = \langle(x,z^2)\rangle$ =

$\langle(x,z^3)\rangle = ... = \langle(x,z^{p-1})\rangle$

(i)If g=(1,I) or g=(1,z) or g=$(1,z^2)$,..or g=$(1,z^{p-1})$ and g∈H

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(1,I)|}\cdot\varphi(g)$$

$$= \frac{p.2^{h+2}}{|C_H(1,I)|} = \frac{p.|C_{Q_{2^{h+1}}}(1)|}{p.|C_{\langle x\rangle}(1)|}.\varphi(1) = \Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z),$(1,z^2)$,….,$(1,z^{p-1})$}

(ii)if g= $(1,I) or\ g=(1,z),..or g=(1,z^{p-1}) or\ g=(x^{2^h},I) or\ g=(x^{2^h},z) or,..g=(x^{2^h},z^{p-1})$,$g\in H$

(iii) if g=(1,I) or g=(1,z) or g=$(1,z^2)$,….or g=$(1,z^{p-1})$

$$\Phi_{(j,2)}(g) = \frac{|C_{Q_{2^{h+1}}\times C_p}(g)|}{|C_H(g)|}\cdot\varphi(g) =$$

$$\frac{p.2^{h+2}}{|C_H(g)|}.1 = \frac{p.2^{h+2}}{|C_{\langle(x,z)\rangle}(g)|}.1 = \frac{p.|C_{Q_{2^{h+1}}}(1)|}{p.|C_{\langle x\rangle}(1)|}.\varphi(1) = \Phi_j(1)$$

since H ∩CL(g)={g}, φ (g)=1

if g=($(x^{2^h},I) or\ g=(x^{2^h},z).....or g=(x^{2^h},z^{p-1}), g\in H$

$$\Phi_{(j,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{p.2^{h+2}}{\left|C_H(g)\right|}.1=\frac{p.\left|C_{Q_{2^{h+1}}}(x^{2^h})\right|}{p.\left|C_{\langle x\rangle}(x^{2^h})\right|}.\varphi(x^{2^h})=\Phi_j(x^{2^h})$$

since H ∩CL(g)={g}, φ (g)=1

(iii) if {g≠ $(x^{2^h},I) or\ g\neq(x^{2^h},z)\}...or g\neq(x^{2^h},z^{p-1}), g\in H$

$$\Phi_{(j,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{p.2^{h+1}}{\left|C_H(g)\right|}(1+1)=$$

$$\frac{p.\left|C_{Q_{2^{h+1}}}(q)\right|}{p.\left|C_{\langle x\rangle}(q)\right|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,

g=(q,z),q∈Q_2^{h+1} and q≠ x^{2^h}

(iv) if g∉H

$\Phi_{(j,2)}(g)=0$ Since H∩CL(g)= ϕ

2- IF H=$\langle(y,I)\rangle$

={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z),…

…(1,z^{p-1}),(y,z^{p-1}),(y^2,z^{p-1}),(y^3,z^{p-1})}

(i) If g=(1,I) or g=(1,z)….or g=(1,z^{p-1})

H∩CL(g)={(1,I),(1,z),….(1,z^{p-1})}

$$\Phi_{(l+1,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{4p.2^h}{4p}.1=2^h=.\Phi_{l+1}(1)$$

(ii) If g= (x^{2^h},I)=(y^2,I),or g=(y^2,z) .or g=(y^2,z^{p-1})

and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{4p.2^h}{4p}.1=2^h=\Phi_{l+1}(x^{2^h})$$

Since H∩CL(g)={g} , φ (g)=1

(iii) If g≠ (x^{2^h},I) and g∈

H,i.e.{g=(y,I),(y,z),…(y,z^{p-1}) or g=(y^3,I),(y^3,z),...(y^3,z^{p-1})}

$$\Phi_{(l+1,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$=\frac{4p}{4p}.(1+1)=2=\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)=ϕ

3.IF H=$\langle(xy,I)\rangle$ =

{(1,I),(xy,I),(($xy)^2$,I)=(y^2,I),(($xy)^3$,I)=(xy^3,I),(1,z),(xy,z),(($xy)^2$,z),(($xy)^3$,z),..,(1,z^{p-1}),(xy,z^{p-1}),(($xy)^2$,z^{p-1}),(($xy)^3$,z^{p-1})}

(i)If g=(1,I) or g=(1,z) …..or g=(1,z^{p-1}) H∩CL(g)={g}

$$\Phi_{(l+2,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{4p.2^h}{4p}.1=2^h=\Phi_{l+2}(1)$$

(ii)If g= (x^{2^h},I)=(($xy)^2$,I)=(y^2,I),(($xy)^2$,z),.,(($xy)^2$,z^{p-1})

and g∈H

$$\Phi_{(l+2,2)}(g)=\frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot\varphi(g)$$

$$=\frac{4p.2^h}{4p}.1=2^h=\Phi_{l+2}(x^{2^h})$$

Since H∩CL(g)={g} , φ (g)=1

(ii) If g≠ (x^{2^h},I) and g∈H

,i.e.g={(xy,I),(($xy)^3$,I),(xy,z),(($xy)^3$,z),.(xy,z^{p-1}),

(iii) $((xy)^3, z^{p-1})\}$

$$\Phi_{(l+2,2)}(g) = \frac{\left|C_{Q_{2^{h+1}}\times C_p}(g)\right|}{\left|C_H(g)\right|}\cdot(\varphi(g)+\varphi(g^{-1}))$$

$$= \frac{4p}{4p}.(1+1) = 2 = \Phi_{l+2}(xy)$$

since $H \cap CL(g) = \{g, g^{-1}\}$ and $\varphi(g) = \varphi(g^{-1}) = 1$

Otherwise

$Ar(Q_{2^5} \times C_7) =$

$\Phi_{(l+2,2)}(g) = 0$ since $H \cap CL(g) = \phi$

Example(4.2):

To construct $Ar(Q_{32} \times C_7)$ by using the theorem (4.1) we get the following table:

Γ- classes	[1,I]	$[x^{16},I]$	$[x^8,I]$	$[x^4,I]$	$[x^2,I]$	[x,I]	[y,I]	[xy,I]	[1,z]	$[x^{16},z]$	$[x^8,z]$	$[x^4,z]$	$[x^2,z]$	[x,z]	[y,z]	[xy,z]
$\lvert CL_\alpha \rvert$	**1**	**1**	**2**	**2**	**2**	**2**	**16**	**16**	**1**	**1**	**2**	**2**	**2**	**2**	**16**	**16**
$\lvert C_{Q_{2^5}\times C_7}(CL_\alpha) \rvert$	**448**	**448**	**224**	**224**	**224**	**224**	**28**	**28**	**448**	**448**	**224**	**224**	**224**	**224**	**28**	**28**
$\Phi_{(1,1)}$	**448**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(2,1)}$	**224**	**224**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(3,1)}$	**112**	**112**	**112**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(4,1)}$	**56**	**56**	**56**	**56**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(5,1)}$	**28**	**28**	**28**	**28**	**28**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(6,1)}$	**14**	**14**	**14**	**14**	**14**	**14**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(7,1)}$	**112**	**112**	**0**	**0**	**0**	**0**	**14**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(8,1)}$	**112**	**112**	**0**	**0**	**0**	**0**	**0**	**14**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(1,2)}$	**64**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**64**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(2,2)}$	**32**	**32**	**0**	**0**	**0**	**0**	**0**	**0**	**32**	**32**	**0**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(3,2)}$	**16**	**16**	**16**	**0**	**0**	**0**	**0**	**0**	**16**	**16**	**16**	**0**	**0**	**0**	**0**	**0**
$\Phi_{(4,2)}$	**8**	**8**	**8**	**8**	**0**	**0**	**0**	**0**	**8**	**8**	**8**	**8**	**0**	**0**	**0**	**0**
$\Phi_{(5,2)}$	**4**	**4**	**4**	**4**	**4**	**0**	**0**	**0**	**4**	**4**	**4**	**4**	**4**	**0**	**0**	**0**
$\Phi_{(6,2)}$	**2**	**2**	**2**	**2**	**2**	**2**	**0**	**0**	**2**	**2**	**2**	**2**	**2**	**2**	**0**	**0**
$\Phi_{(7,2)}$	**16**	**16**	**0**	**0**	**0**	**0**	**2**	**0**	**16**	**16**	**0**	**0**	**0**	**0**	**2**	**0**
$\Phi_{(8,2)}$	**16**	**16**	**0**	**0**	**0**	**0**	**0**	**2**	**16**	**16**	**0**	**0**	**0**	**0**	**0**	**2**

Table (5)

Proposition(4.3):

If m=2^{h} ,h any positive integer and p is prime number ,

then the matrix M($Q_{2m}\times C_p$) of the group ($Q_{2m}\times C_p$) is :

$$M(Q_{2m}\times C_p)=\left[\begin{array}{c|c} M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) \end{array}\right]$$

Which is 2(h+4)×2(h+4) square matrix ,M(Q_{2m}) is similar to the matrix in Proposition (3.7).

Proof :

By Proposition (4.1) we obtain the Artin's characters Table Ar($Q_{2m}\times C_p$) of the group ($Q_{2m}\times C_p$) when $m=2^h, h\in Z^+$ and p is prime number and from the theorem (1.7) we get the rational valued characters $(\overset{*}{\equiv}(Q_{2m}\times C_p))$ table of the group ($Q_{2m}\times C_p$) when $m=2^h, h\in Z^+$ and p is prime number.

Thus, by definition of M(G) we can find the matrix M($Q_{2m}\times C_p$) when $m=2^h, h\in Z^+$ and p is prime number.

$$M(Q_{2m}\times C_p)=Ar(Q_{2m}\times C_p)\cdot(\overset{*}{\equiv}(Q_{2m}\times C_p))^{-1}.$$

$$=\left[\begin{array}{cccccccccccccccccccc}
2&2&2&\cdots&\cdots&2&1&1&1&1&2&2&2&\cdots&\cdots&2&1&1&1&1\\
0&2&2&\cdots&\cdots&2&1&1&1&1&0&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&2&\cdots&\cdots&2&1&1&1&1&0&0&2&\cdots&\cdots&2&1&1&1&1\\
\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots\\
0&0&0&\cdots&\cdots&2&1&1&1&1&0&0&0&\cdots&\cdots&2&1&1&1&1\\
0&0&0&\cdots&\cdots&0&1&1&1&1&0&0&0&\cdots&\cdots&0&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&1&0&1&0&0&0&\cdots&\cdots&0&0&1&0&1\\
0&1&1&\cdots&\cdots&1&0&0&1&1&0&1&1&\cdots&\cdots&1&0&0&1&1\\
0&1&1&\cdots&\cdots&1&1&0&0&1&0&1&1&\cdots&\cdots&1&1&0&0&1\\
0&0&\cdots&&\cdots&&\cdots&&0&0&2&2&2&\cdots&\cdots&2&1&1&1&1\\
0&0&\cdots&&\cdots&&\cdots&&0&0&0&2&2&\cdots&\cdots&2&1&1&1&1\\
&&\ddots&&&&&&&&0&0&2&\cdots&\cdots&2&1&1&1&1\\
\vdots&\vdots&&&&&&&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&&&\ddots&&&&\vdots&\vdots&\vdots&\vdots&\vdots&&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&&&&&&&\vdots&\vdots&0&0&0&\cdots&\cdots&2&1&1&1&1\\
&&&&&&&&&&0&0&0&\cdots&\cdots&0&1&1&1&1\\
&&&&&&&\ddots&&&0&0&0&\cdots&\cdots&0&0&1&0&1\\
0&0&\cdots&&\cdots&&\cdots&&0&0&0&1&1&\cdots&\cdots&1&0&0&1&1\\
0&0&\cdots&&\cdots&&\cdots&&0&0&0&1&1&\cdots&\cdots&1&1&0&0&1
\end{array}\right]$$

$$=\left[\begin{array}{cccccccccccccccccccc}
&&&&&&1&1&1&1&&&&&&&1&1&1&1\\
&&&&&&1&1&1&1&&&&&&&1&1&1&1\\
&&2R_1(C_{2m})&&&&1&1&1&1&&&2R_1(C_{2m})&&&&1&1&1&1\\
&&&&&&\vdots&\vdots&\vdots&\vdots&&&&&&&\vdots&\vdots&\vdots&\vdots\\
&&&&&&\vdots&\vdots&\vdots&\vdots&&&&&&&\vdots&\vdots&\vdots&\vdots\\
&&&&&&1&1&1&1&&&&&&&1&1&1&1\\
0&0&0&\cdots&\cdots&0&1&1&1&1&0&0&0&\cdots&\cdots&0&1&1&1&1\\
0&0&0&\cdots&\cdots&0&0&1&0&1&0&0&0&\cdots&\cdots&0&0&1&0&1\\
0&1&1&\cdots&\cdots&1&0&0&1&1&0&1&1&\cdots&\cdots&1&0&0&1&1\\
0&1&1&\cdots&\cdots&1&1&0&0&1&0&1&1&\cdots&\cdots&1&1&0&0&1\\
0&0&\cdots&&\cdots&\cdots&0&0&&&&&&&&&1&1&1&1\\
0&0&\cdots&&\cdots&\cdots&0&0&&&&&&&&&1&1&1&1\\
&&\ddots&&&&&&&&&&2R_1(C_{2m})&&&&1&1&1&1\\
\vdots&\vdots&&&&&\vdots&\vdots&&&&&&&&&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&&\ddots&&&\vdots&\vdots&&&&&&&&&\vdots&\vdots&\vdots&\vdots\\
\vdots&\vdots&&&&&\vdots&\vdots&&&&&&&&&1&1&1&1\\
&&&&&&&&&&0&0&0&\cdots&\cdots&0&1&1&1&1\\
&&&&&&&&\ddots&&0&0&0&\cdots&\cdots&0&0&1&0&1\\
0&0&\cdots&&\cdots&\cdots&0&0&0&1&&&1&\cdots&\cdots&1&0&0&1&1\\
0&0&\cdots&&\cdots&\cdots&0&0&0&1&&&1&\cdots&\cdots&1&1&0&0&1
\end{array}\right]$$

$$=\left[\begin{array}{c|c} M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) \end{array}\right]=M(Q_{2m}\times C_p)$$

Example (4.4):

Consider the group ($Q_{32}\times C_7$), we can find the matrix $M(Q_{32}\times C_7)$ by using:

$$M(Q_{32}\times C_7)=M(Q_{2^5}\times C_7)=Ar(Q_{2^5}\times C_7)\cdot(\overset{*}{\equiv}(Q_{2^5}\times C_7))^{-1}$$

$$=\begin{bmatrix}
448&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
224&224&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
112&112&112&0&0&0&0&0&0&0&0&0&0&0&0&0\\
56&56&56&56&0&0&0&0&0&0&0&0&0&0&0&0\\
28&28&28&28&28&0&0&0&0&0&0&0&0&0&0&0\\
14&14&14&14&14&14&0&0&0&0&0&0&0&0&0&0\\
112&112&0&0&0&0&14&0&0&0&0&0&0&0&0&0\\
112&112&0&0&0&0&0&14&0&0&0&0&0&0&0&0\\
64&0&0&0&0&0&0&0&64&0&0&0&0&0&0&0\\
32&32&0&0&0&0&0&0&32&32&0&0&0&0&0&0\\
16&16&16&0&0&0&0&0&16&16&16&0&0&0&0&0\\
8&8&8&8&0&0&0&0&8&8&8&8&0&0&0&0\\
4&4&4&4&4&0&0&0&4&4&4&4&4&0&0&0\\
2&2&2&2&2&2&0&0&2&2&2&2&2&2&0&0\\
16&16&0&0&0&0&2&0&16&16&0&0&0&0&2&0\\
16&16&0&0&0&0&0&2&16&16&0&0&0&0&0&2
\end{bmatrix}\bullet$$

$$\begin{bmatrix}
1/224&1/224&1/224&1/224&1/448&1/448&1/448&1/448&1/224&1/224&1/224&1/224&1/448&1/448&1/448&1/448\\
-1/224&1/224&1/224&1/224&1/448&1/448&1/448&1/448&-1/224&1/224&1/224&1/224&1/448&1/448&1/448&1/448\\
0&-1/112&1/112&1/112&1/224&1/224&1/224&1/224&0&-1/112&1/112&1/112&1/224&1/224&1/224&1/224\\
0&0&-1/56&1/56&1/112&1/112&1/112&1/112&0&0&-1/56&1/56&1/112&1/112&1/112&1/112\\
0&0&0&-1/28&1/56&1/56&1/56&1/56&0&0&0&-1/28&1/56&1/56&1/56&1/56\\
0&0&0&0&-1/28&1/28&-1/28&1/28&0&0&0&0&-1/28&1/28&-1/28&1/28\\
0&0&0&0&-1/28&-1/28&1/28&1/28&0&0&0&0&-1/28&-1/28&1/28&1/28\\
0&0&0&0&1/28&-1/28&-1/28&1/28&0&0&0&0&1/28&-1/28&-1/28&1/28\\
-1/224&-1/224&-1/224&-1/224&-1/448&-1/448&-1/448&-1/448&3/112&3/112&3/112&3/112&3/224&3/224&3/224&3/224\\
1/224&-1/224&-1/224&-1/224&-1/448&-1/448&-1/448&-1/448&-3/112&3/112&3/112&3/112&3/224&3/224&3/224&3/224\\
0&1/112&-1/112&-1/112&-1/224&-1/224&-1/224&-1/224&0&-3/56&3/56&3/56&3/112&3/112&3/112&3/112\\
0&0&1/56&-1/56&-1/112&-1/112&-1/112&-1/112&0&0&-3/28&3/28&3/56&3/56&3/56&3/56\\
0&0&0&1/28&-1/56&-1/56&-1/56&-1/56&0&0&0&-3/14&3/28&3/28&3/28&3/28\\
0&0&0&0&1/28&-1/28&1/28&-1/28&0&0&0&0&-3/14&3/14&-3/14&3/14\\
0&0&0&0&1/28&1/28&-1/28&-1/28&0&0&0&0&-3/14&-3/14&3/14&3/14\\
0&0&0&0&-1/28&1/28&1/28&-1/28&0&0&0&0&3/14&-3/14&-3/14&3/14
\end{bmatrix}$$

$$=\begin{bmatrix}
2&2&2&2&1&1&1&1&2&2&2&2&1&1&1&1\\
0&2&2&2&1&1&1&1&0&2&2&2&1&1&1&1\\
0&0&2&2&1&1&1&1&0&0&2&2&1&1&1&1\\
0&0&0&2&1&1&1&1&0&0&0&2&1&1&1&1\\
0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1\\
0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&1\\
0&1&1&1&0&0&1&1&0&1&1&1&0&0&1&1\\
0&1&1&1&1&0&0&1&0&1&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&2&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&2&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&2&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&1&1\\
0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&1
\end{bmatrix}$$

Proposition(4.5):

If $m=2^h$, h any positive integers and p is prime number then the matrices $P(Q_{2m}\times C_p)$ and $W(Q_{2m}\times C_p)$ are taking the forms :

$$P(Q_{2m}\times C_p)=\left[\begin{array}{c|c} P(Q_{2m}) & -P(Q_{2m}) \\ \hline 0 & P(Q_{2m}) \end{array}\right]$$

which is 2(h+4)×2(h+4) square matrix .

And

$$W(Q_{2m}\times C_p)=\left[\begin{array}{c|c} W(Q_{2m}) & 0 \\ \hline 0 & W(Q_{2m}) \end{array}\right]$$

which is 2(h+4)×2(h+4) square matrix .

Proof :

By using the proposition (4.3) taking the matrix $M(Q_{2m}\times C_p)$ and the above forms $P(Q_{2m}\times C_p)$ and $W(Q_{2m}\times C_p)$ then we have :

$P(Q_{2m}\times C_p).\ M(Q_{2m}\times C_p).W(Q_{2m}\times C_p)=$

$$\text{diag }\{\underbrace{2,2,2,2,\cdots,2}_{2(h+1)},1,1,1,1,1,1\}$$

$=D(Q_{2m}\times C_p)$

which is 2(h+4)×2(h+4) square matrix .

Example(4.6):

To find the matrices $P(Q_{32} \times C_7)$ and $W(Q_{32} \times C_7)$ by the proposition (4.5) from Example (3.9) to find $P(Q_{32})$ and $W(Q_{32})$:

$$P(Q_{32} \times C_7) = \left[\begin{array}{c|c} P(Q_{32}) & -P(Q_{32}) \\ \hline 0 & P(Q_{32}) \end{array}\right] =$$

$$\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

And

$$W(Q_{32} \times C_7) = \left[\begin{array}{c|c} W(Q_{32}) & 0 \\ \hline 0 & W(Q_{32}) \end{array}\right] =$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1
\end{bmatrix}$$

Example(4.7)

To find $D(Q_{32} \times C_7)$ and the cyclic decomposition of the factor group

We find the matrices $P(Q_{32} \times C_7)$ and $W(Q_{32} \times C_7)$ as in example (4.6) and $M(Q_{32} \times C_7)$ as in example (4.4),then :

$P(Q_{32} \times C_7).M(Q_{32} \times C_7).W(Q_{32} \times C_7)=$

$\mathrm{diag}\{2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1\}=D(Q_{32} \times C_7)$

Then by Theorem (3.2) we have

$$AC(D(Q_{32} \times C_7)= \bigoplus_{i=1}^{10} C_2$$

The following theorem gives the cyclic decomposition of the factor group $AC(D(Q_{2m} \times C_7))$ when $m=2^h$, $h \in Z^+$ and p is prime number .

Theorem(4.8):

If $m= 2^h$, h any positive integer and p is prime number then the cyclic decomposition of $AC(Q_{2m} \times C_p)$ is :

$$AC(D(Q_{2m} \times C_p))= \bigoplus_{i=1}^{2(h+1)} C_2$$

Proof :

By using the proposition (4.3),we can find matrix $M(Q_{2m} \times C_p)$ and by the proposition (4.5),we find $P(Q_{2m} \times C_p)$ and $W(Q_{2m} \times C_p)$ when $m=2^h$, $h \in Z^+$ and p is prime number :

$P(Q_{2m} \times C_p).\ M(Q_{2m} \times C_p).\ W(Q_{2m} \times C_p)=$

$\mathrm{diag}\{2,2,2,2,2,2,\ldots,2,2,2,1,1,1,1,1,1\}$

Then ,by the theorem (3.2) we have :

$$AC(D(Q_{2m} \times C_p))= \bigoplus_{i=1}^{2(h+1)} C_2$$

Example(4.9) :

Consider the groups ($Q_{16384}\times C_{11}$) ,($Q_{134217728}\times C_5$),then :

1. $AC(Q_{16384}\times C_{11}) = AC(Q_{2^{14}}\times C_{11}) = \bigoplus_{i=1}^{30} C_2$

2. $AC(Q_{134217728}\times C_5) = AC(Q_{2^{27}}\times C_5) = \bigoplus_{i=1}^{56} C_2$

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التجزئة الدائرية وجدول شواخص ارتن للزمرة $(Q_{2m} \times C_p)$ عندما $h \in Z^+, m=2^h$ و p عدد اولي

المدرس المساعد رجاء حسن عباس

Email: rajaah.alabidy@uokufa.edu.iq

جامعة الكوفه - كلية التربية للبنات- قسم الرياضيات

المستخلص :

الهدف الأساسي من هذا البحث هو تحديد ألتجزئه الدائرية للزمرة الابيليه الكسريه المنتهية $AC(G) = \overline{R}(G)/T(G)$ حيث ان $G=Q_{2m}\times C_p$ عندما $h \in Z^+, m=2^h$ و p عدد أولي (زمرة كل الشواخص العمومية ذات القيم الصحيحة للزمرة G على زمرة الشواخص المحتثة من الشواخص الأحادية للزمر الجزئية الدائرية). وجدنا ان التجزئة الدائرية للزمرة $AC(Q_{2m}\times C_p)$ تعتمد على القواسم الأولية للعدد m حيث انه اذا كان $m=2^h$، h عدد صحيح موجب و p عدد أولي فان:

$$AC(Q_{2m}\times C_p) = \bigoplus_{i=1}^{2(h+1)} C_2$$

كذلك وجدنا الصيغة العامة لجدول شواخص آرتن $Ar(Q_{2m}\times C_p)$ عندما $h \in Z^+, m=2^h$ و p عدد أولي