On s-coc-separation axioms

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Abstract
In this paper we introduced new type of separation axioms called s-coc-separation axiomsand we introducedtypes of continuous functionsand study the relation among them. We studiedthe definitions of s-coc-separation axioms ,properties and relation among them .

Keywords
S-coc-open set , locally s-coc-closed set , s-coc-continuous function , s-coc´-continuous function, **s**-coc-T_i for i= 1,2 ,s-coc-regular **,** s-coc-normal , s-coc*-regular , s-coc*-normal , locally s-coc-regular , locally s-coc-normal **,** locally s-coc* -regular , locally s-coc* -normal **.**

Introduction
This paper consist of three sections . In section one we study definition of s-coc-open set and locally s-coc-closed set and study some properties . In section two study s-coc-continuous , s-coc´-continuous and we prove propositions . In section three we defined new types of s-coc-separation axioms and we introduced relation among them .

Definition : - (1.1) [5]
A subset A of a space $(X, \tau)$is called a cocompact open set (coc-open-set) if every $x \in A$ there exists open set $U \subseteq X$ and compact subset K such that $x \in U - K \subseteq A$, the complement of coc-open set is called coc-closed set .

Definition :- (1.2) [7]
Asubset A of space X is called a semi open set (s-open) if and only if $A \subseteq \overline{A^\circ}$ and Ais called s-closed if and only if A^c s-open.

Proposition (1.1)[6]
For any subset A of space X the following statements are equivalent .
1) A is s-open set .
2) $\bar{A} = \overline{A^\circ}$
3) There exists open set G such that $G \subseteq A \subseteq \overline{G}$

Remark (1.1) [7]
Every open set is semi-open .but the convers is not true

Proposition (1.2) [2]
For any subset A of a space X the following statements are equivalent
1. A is s-closed
2. $A^{\circ} = \overline{A}^{\circ}$
3. There exists closed set F in X such that $F^{\circ} \subseteq A \subseteq F$

Definition (1.3)
A subset A of a space (X, τ) is called semi-cocompact open set (s-coc-open-set) if for every $x \in A$ there exists s-open set $U \subseteq X$ and compact subset K such that $x \in U - K \subseteq A$, the complement of s-coc-open set is called s-coc-closed set .

Remark (1.2)
Every coc-open set is s-coc-open set .but the convers is not true for the following example :-

Example (1.1)
Consider the space $X = \{1,2,3,4, \dots\}$, $\tau = \{\emptyset, X, \{2\}, \{3\}, \{2,3\}\}$ topology on X, $A = \{1,2\}$ s-coc-open set but not coc-open set.

Remark (1.3) :-
i- Every open set is s-coc-open set .
ii- Every s-open set is s-coc-open set .

proof:
i. Let A open set. Then A s-open and K compact.Then for all $x \in A$ we have $x \in A - K \subseteq A$.
ii.Clear .
but the convers is not true for the following example :-
Let $X = \{1,2,3,4, \dots\}$, $\tau = \{\emptyset, X, \{2\}\}$. It is clear that $\{1\}$ s-coc-open set but not open and not s-open set .

Remark (1.4) :-
1. The intersection of open set and s-open is s-open [7] .
2. The intersection of two s-coc-open is s-coc-open set .
3. The intersection of s-coc-open and coc-open set is s-coc-open
4. The union of s-coc-open is s-coc-open set
5. The intersection of s-coc-open sets and open set is s-coc-open

Proof: -
2. Let A and B s-coc-open sets. To prove $A \cap B$ is s-coc-open set. And let $A \cap B$ is not s-coc- open set.Then there exists $x \in A \cap B$ such that for all V_x s-open set and Kcompact $x \in V_x - K \nsubseteq A \cap B$. Then $x \in V_x - K \nsubseteq A$ or$x \in V_x - K \nsubseteq B$. Then Ais not s-coc orB is not s-coc-open set.This contradicationsinceA , B s-coc-open sets. Then $A \cap B$ is s-coc-open set.

3. Let A s-coc-open set and B coc-open set .sinceB coc-open then B s-coc-open. Then $A \cap B$ is s-coc-open set by (2)

4. $\{A_\alpha : \alpha \in \Lambda\}$s-coc-open set .let $x \in \cup A_\alpha$ .Then$x \in A_\alpha$ for some $\alpha \in \Lambda$.Thenthere exists U_α s-open set and K_α compact such that$x \in U_\alpha - K_\alpha \subseteq A_\alpha \subseteq \cup A_\alpha$ then$x \in U_\alpha - K_\alpha \subseteq \cup A_\alpha$,then $\cup A_\alpha$ s-coc-open set .

5.Let A s-coc-open set and B open set .Then for all $x \in A$ there exists U s-open set and K compact such that $x \in U - K \subseteq A$, since U s-open and B open ,then $A \cap B$ s-open by (1),then $x \in (U - K) \cap B \subseteq A \cap B$ then $x \in U - K^c \cap B \subseteq A \cap B$ then$x \in (U \cap B) - K \subseteq A \cap B$then $A \cap B$ s-coc-open set.

Remark (1.5) :-

1. The coc-open sets forms topology on X denoted by τ^k [9]
2. The s-coc-open sets forms topology on X denoted by τ^{sk}
3. Every s-closed is s-coc -closed but the converse is not true for example

Example(1.2)

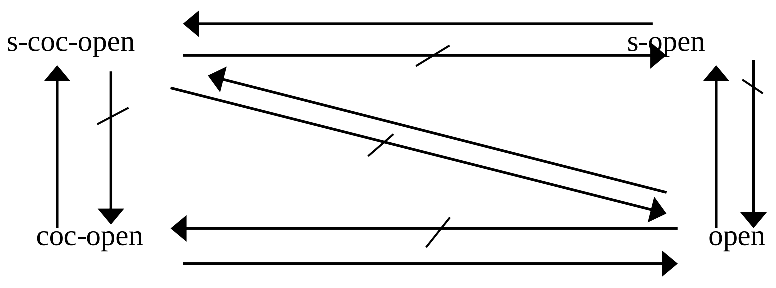
Let $X = \{1,2,3,4, \ldots .\}$, $\tau = \{\emptyset, X, \{2\}, \{3\}, \{2,3\}\}$ topology on X and $A = \{1,2\}$s-coc-open set then$A^c = \{3,4,5,6, \ldots .\}$ s-closed but A^c not s-closed .

Proposition (1.3)

Let X and Y be topological spacesand $A \subseteq X$, $B \subseteq Y$such that A s-coc-open set in X and B s-coc-open set in Y then $A \times B$ is s-coc-open subset in $X \times Y$.

Proof :-

Let$(x, y) \in A \times B$, then $x \in A$ and $y \in B$ .Since A is s-coc-open in X . Then for all $x \in A$ there exists U s-open set and K_1 compact such that $x \in U - K_1 \subseteq A$. Since B is s-coc-open in Y . Then for all $y \in B$ there exists V s-open set and K_2compact such that $y \in V - K_2 \subseteq B$ Since U and V are s-open sets ,Then $U \subseteq \overline{U^\circ}$ and $V \subseteq \overline{V^\circ}$ then $U \times V \subseteq \overline{U^\circ} \times \overline{V^\circ} \subseteq \overline{U^\circ \times V^\circ} \subseteq \overline{(U \times V)^\circ}$.Then $U \times V \subseteq \overline{(U \times V)^\circ}$, then$U \times V$ s-open in $X \times Y$and$K_1 \times K_2$ compact in $X \times Y$ .Then for all $(x, y) \in A \times B$ there exists s-open $U \times V = W$ and $K_1 \times K_2 = K$ compat such that $(x, y) \in W - K \subseteq A \times B$ therefor $A \times B$ s-coc-open in $X \times Y$.



Definition (1.4)

Let X be space and A ⊆ X. The intersection of all s-coc-closed setsX containing A called the s-coc- closure of A defined by .$\overline{A}^{s-coc}$ $=\cap\{B: B\ s-coc-\text{closed in X and } A \subseteq B\}$

Definition (1.5)[5]

Let X be space and A ⊆ X. The intersection of all coc-closed sets X containing A called the coc-closure of A defined by$\overline{A}^{coc}$ $=\cap\{B: B\ \ coc-\text{closed in X and } A \subseteq B\}$

Proposition (1.4)

Let X be a topological space and A ⊆ Xthen $\overline{A}^{s-coc}$ is the smallest s-coc-closed set containing A.

Proof

Clear .

Proposition (1.5)[5]

Let X be a topological space and A ⊆ X ,thenx ∈ $\overline{A}^{coc}$ if and only if for each coc-open in X contained point x we have U ∩ A ≠ ∅ .

Proposition (1.6)

Let X be a topological space and A ⊆ X , then x ∈ $\overline{A}^{s-coc}$if and only if for each s-coc-open in X contained point x we have U ∩ A ≠ ∅ .

Proof :

Assume that x ∈ $\overline{A}^{s-coc}$and let U s-coc-open in X such that x ∈ U , and suppose U ∩ A ≠ ∅thenA ⊆ U^c. Since$U$ s-coc-open set in X and x ∈ U then U^c s-coc closed set in X and x ∉ Uand $\overline{A}^{s-coc}$is smallest s-coc-closed contain A then$\overline{A}^{s-coc}$ ⊆ U^c . Since U ∩ U^c = ∅ and x ∈ U then x ∉ U^c then x ∉ $\overline{A}^{s-coc}$.

Conversely :-

Let U s-coc-closed set in X such that x ∈ U and U ∩ A ≠ ∅.To provex ∈ $\overline{A}^{s-coc}$.Let x ∉ $\overline{A}^{s-coc}$ then x ∈ $(\overline{A}^{s-coc})^c$, since $\overline{A}^{s-coc}$ is s-coc-closed in X , $(\overline{A}^{s-coc})^c$ is s-coc-open in Xand $\overline{A}^{s-coc} \cap (\overline{A}^{s-coc})^c = \emptyset$ .Then $A \cap (\overline{A}^{s-coc})^c = \emptyset$,since$A \subseteq (\overline{A}^{s-coc})^c$ .This is a contradiction since for every s-coc-open set$U$in X , U ∩ A ≠ ∅ .

Proposition (1.7)

Let X be a topological space and A ⊆ B then

i- $(\overline{A}^{s-coc})^c$ is s-coc-closed set

ii- A is s-coc-closed if and only if $A = \overline{A}^{s-coc}$

iii- $\overline{A}^{s-coc} = \overline{\overline{A}^{s-coc}}^{s-coc}$

iv- If A ⊆ B then $\overline{A}^{s-coc} \subseteq \overline{B}^{s-coc}$

v- $\overline{A}^{s-coc} \subseteq \overline{A}$

vi- $\overline{A}^{s-coc} \subseteq \overline{A}^{coc}$

Proof :-

i- By definition of s-coc-closed set.

ii- Let A is s-coc-closed in X . Since $A \subseteq \overline{A}^{s-coc}$ and $\overline{A}^{s-coc}$ smallest s-coc-closed set containing A , then $\overline{A}^{s-coc} \subseteq A$ then $A = \overline{A}^{s-coc}$

conversely :-

Let $A = \overline{A}^{s-coc}$. Since $\overline{A}^{s-coc}$ is s-coc-closed then A is s-coc-closed.

iii- From (i) and (ii)

iv- Let $A \subseteq B$. Since $B \subseteq \overline{B}$ then $A \subseteq \overline{A}^{s-coc}$. Since $\overline{A}^{s-coc}$ smallest s-coc-closed set containing A then $\overline{A}^{s-coc} \subseteq \overline{B}^{s-coc}$

v- Let $x \in \overline{A}^{s-coc}$ then for all s-coc-open set U such that $x \in U$ we have $U \cap A \neq \emptyset$ .Then for all open set U such that $x \in U$ we have $U \cap A \neq \emptyset$ by proposition (1.5). Then $x \in \bar{A}$.

vi- by proposition(1.6) and proposition(1.5).

Definition (1.6)

Let X be space and $A \subseteq X$. The union of all s-coc-open sets of X containing in A is called s-coc-Interior of A denoted by $A^{\circ s-coc} = \cup \{B : B \ s-coc-open \text{ in X and } B \subseteq A\}$

Definition (1.7)[5]

Let X be space and $A \subseteq X$. The union of all coc-open sets of X containing in A is called coc-Interior of A denoted by

$A^{\circ coc} = \cup \{B : B \ \ coc-open \text{ in X and } B \subseteq A\}$

Proposition (1.8):-

Let X be a topological space and $A \subseteq X$, then $A^{\circ s-coc}$ is the largest s-coc-open set contain A

Proof :

Clear.

Proposition(1.9)

Let X be a topological space and $A \subseteq X$, then $x \in A^{\circ s-coc}$ if and only if there exists s-coc-open set V containing x such that $x \in V \subseteq A$.

Proof :

Let $x \in A^{\circ s-coc}$ then $x \in \cup U$ such that U s-coc-open set and $x \in V \subseteq A$.

Conversely

Let there exists V s-coc-open set such that $x \in V \subseteq A$ then $x \in \cup V$, $V \subseteq A$ and V s-coc-open set then $x \in A^{\circ s-coc}$.

Proposition(1.10)[5]

Let X be a topological space and $A \subseteq X$, then $x \in A^{\circ coc}$ if and only if there exists coc-open set V containing x such that $x \in V \subseteq A$.

Proposition (1.11)

Let X be a topological space and $A \subseteq B \subseteq X$then .
1. $A^{\circ s-coc}$is s-coc-open set .
2. Ais s-coc-open if and only if $A = A^{\circ s-coc}$.
3. $A^{\circ} \subseteq A^{\circ s-coc}$.
4. $A^{\circ s-coc} = (A^{\circ s-coc})^{\circ s-coc}$.
5. if$A \subseteq B$ then $A^{\circ s-coc} \subseteq B^{\circ s-coc}$.
6. $A^{\circ coc} \subseteq A^{\circ s-coc}$

Proof :-

1. and 2. from definition (1.5)

3. Let$x \in A^{\circ s-coc}$then there exists U open set such that $x \in U \subseteq A$ then U s-coc-open set then U s-coc-open set such that $x \in U \subseteq A$ thus $x \in A^{\circ s-coc}$

4.from (1) and (2).

5.Let$x \in A^{\circ s-coc}$ then there exists Vs-coc-open set such that $x \in V \subseteq A$ by proposition(1.9), since $A \subseteq B$ then $x \in V \subseteq B$ .Then $x \in B^{\circ s-coc}$ by proposition(1.9) . Thus $A^{\circ s-coc} \subseteq B^{\circ s-coc}$.

6. By proposition(1.10) and proposition (1.9)

Proposition (1.12)

Let X be a space and $A \subseteq X$, then $(A^c)^{\circ s-coc} = (\overline{A}^{s-coc})^c$

Proof

Let $x \in (A^c)^{\circ s-coc}$ and $x \notin (\overline{A}^{s-coc})^c$. Then $x \in \overline{A}^{s-coc}$.Then for all $x \in A$ there exists U s-coc-open setsuch that $U \cap A \neq \emptyset$ by proposition (1.6) . Since $(A^c)^{\circ s-coc}$ s-coc-open set then $(A^c)^{\circ s-coc} \cap A \neq \emptyset$ .Then $(A^c)^{\circ s-coc} \subseteq A^c$ then$A \cap A^c \neq \emptyset$.This is contradiction Thus $x \in (\overline{A}^{s-coc})^c$ .Then $(A^c)^{\circ s-coc} \subseteq (\overline{A}^{s-coc})^c$, $l$et $x \in (\overline{A}^{s-coc})^c$ then $x \notin \overline{A}^{s-coc}$. Then there exists U s-coc-open set such that $U \cap A \neq \emptyset$. Then $U \subseteq A^c$ Therefore$U^{\circ s-coc} \subseteq (A^c)^{\circ s-coc}$ .Thus $x \in (A^c)^{\circ s-coc}$ then $(A^c)^{\circ s-coc} = (\overline{A}^{s-coc})^c$.

Definition (1.8):-[1]

Let X be a space and B any subset of X, a neighborhood of B is any subset of X which contains an open set containing B . The neighborhoods of a subset $\{x\}$ is also neighborhood of the point x .

Remark (1.6)

The collection of all neighborhoods of the subset B of X are denoted by $N(B)$. In particular the collection of all neighborhoods of x is denoted by $N(x)$.

Definition (1.9)

Let X be a space and B$\subseteq X$, an s-coc-neighborhood of B is any subset of X which contains an s-coc-open set containing B .The s-coc-neighborhood of subset $\{x\}$ is also called s-coc-neighborhood of the point x.

Remark (1.7)

The collection of all neighborhoods of the subset B of X are denoted by $N_{s-coc}(B)$ in particular the collection of all neighborhoods of x is denoted by $N_{s-coc}(x)$.

Proposition (1.13)

Let (X,τ) be a topological space and for each $\in X$, let $N_{s-coc}(x)$ be a collection of all s-coc-neighborhoods of x then :-

i. If $A \in N_{s-coc}(x)$ such that $A \subseteq B$ then $B \in N_{s-coc}(x)$
ii. If $A, B \in N_{s-coc}(x)$ then $A \cap B \in N_{s-coc}(x)$ such that $A, B \subseteq X$
iii. If $A_\alpha \in N_{s-coc}(x)$ then $\bigcup A_\alpha \in N_{s-coc}(x)$

Proof

i- Since $A \in N_{s-coc}(x)$ then there exists U s-coc-open set such that$x \in U \subseteq A$, since $A \subseteq B$ then $x \in U \subseteq B$hence $B \in N_{s-coc}(x)$.

ii- Let$A, B \in N_{s-coc}(x)$ and $A \cap B \notin N_{s-coc}(x)$ .Then$x \in A \cap B$ and for all U s-coc-open set such that x $\in U \nsubseteq A \cap B$, $x \in U \nsubseteq A or x \in U \nsubseteq B$ .Then$A \notin N_{s-coc}(x) or B \notin N_{s-coc}(x)$this contradiction .

iii- Since$A_\alpha \in N_{s-coc}(x)$ exists U_α s-coc-open set such that$x \in U_\alpha \subseteq A_\alpha \subseteq \bigcup A_\alpha$ .Then$x \in U_\alpha \subseteq \bigcup A_\alpha$. Therefore $\bigcup A_\alpha \in N_{s-coc}(x)$.

Proposition (1.14)

Let (X,τ) be a space and $A \subseteq X$ then A s-coc-open set in X if and only if A is s-coc-neighborhood for all his points in A

Proof

Let A s-coc-open and $x \in A$ .Since $x \in A \subseteq A$ then A is s-coc- neighborhood of x for all x hence$A$ is s-coc- neighborhood for all his points

Conversely :

Let$A$ is s-coc- neighborhood for all his points and$x \in A$ .Then$A$ is s-coc- neighborhood for $x$then there exists U_x s-coc-open set such that $x \in U_x \subseteq A$ .Then $A = \bigcup\{x : x \in A\} \subseteq \{U_x : x \in U_x\} \subseteq A$ .Then $A = \{U_x : x \in U_x\}$ .Then A union of s-coc-open sets . Therefor $A$is s-coc-open set

Proposition (1.15)

If $X\ is\ a\ discrete\ space\ then\ N_{s-coc}(x) = \{A : x \in A\}$

Proof

Since τ discrete topologythen $\tau = \{A : x \in A\}$. Let $A \subseteq X$ and $x \in X$ either $x \in A or x \notin A$

-if$x \notin A$ then Anot s-coc- neighborhood of x hence$A \notin N_{s-coc}(x)$

-if $x \in A$ .Since$\{x\} \subseteq X$, and $\{x\}$ open set in X . Then$\{x\}$ s-coc-open set $x \in \{x\} \subseteq A$ then$A$ s-coc- neighborhood of x then$A \in N_{s-coc}(x)$hence$N_{s-coc}(x) = \{A : x \in A\}$

Remark (1.8)

Every neighborhood of x is s-coc- neighborhood of x . But the converse not true for example

Example (1.3)

Let $X = \{1,2,3,4\} and \tau = \{\emptyset, X, \{4\}\}$, $A = \{1,2\}$not neighborhood of x but s-coc-neighborhood of x

Definition (1.10)[4]

Let (X, τ) be a topological space and A subset of X is called a locally closed set if A= $U \cap F$, $w$here U ∈ $\tau$and F closed in X.

Definition (1.11)

Let (X, τ) be a topological space and A subset of X is called a locally s-coc-closed set if A= $U \cap F$, where $U \in \tau$, F s-coc-closed in X

Proposition (1.16)

Every locally closed set is locally s-coc-closed.

Proof

Let A be locally closed then$A = U \cap F$ such that $U \in \tau$, F closed set . Since every closed is s-coc-closed then A is locally s-coc-closed set.But the convers is not true for the following example.

Example (1.4)

Let $X = \{1,2,3, \ldots .\}$, $\tau = \{\emptyset, X, \{1,2,3\}\}$ and $A = \{1,2\}$.Since $A = \{1,2,3\} \cap \{1,2,5\}$ and $\{1,2,5\}$ s-coc-closed .Then A is locally s-coc-closed set . But there exist no F closed set such that $A = \{1,2,3\} \cap F$.

Proposition (1.17)

i. The intersection of two locally s-coc-closed set is locally s-coc-closed set
ii. The union of any family of locally s-coc-closed sets is locally s-coc-closed

Proof

i. Let A, B are locally s-coc-closed sets. Then $A = U \cap F$, $B = V \cap L$,suchthat$U, V \in \tau$ and F, L are s-coc-closed sets in X .Then $F \cap L = M$ is s-coc-closed sets in X and U∩V∈ τ . Then $A \cap B$ locally s-coc-closed sets.

ii. Let $\{A_\propto : \propto \in \Lambda\}$ family of locally s-coc-closed sets .
Then $\cup A_\propto = \cup (U_\propto \cap F_\propto) = \cup U_\propto \cap (\cup F_\propto) = W_\propto \cap L_\propto$
Since $U_\propto \in \tau$then $\cup U_\propto \in \tau$. Since $F_\propto$ s-coc-closed set in X .Then $L_\propto = \cup F_\propto$ s–coc–closed set .Then $\cup A_\propto$ locally s-coc-closed set .

Proposition(1.18)
If X space and τ discrete topology on X then any subset of X is locally s-coc-closed set.
proof:
Let A$\subseteq$ X, since τ is discrete topology on X . Then A is open and closed set .Since A$= A \cap A$ then A is locally closed set .Then A is locally s-coc closed set byproposition(1.16).

2.On s-coc- continuous function

In this section, we introduce the definition of s-coc-continuous , remarks and propositions about this concept .

Definition (2.1)[1]
Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y.Then f is called a continuous function if $f^{-1}(A)$ is open set in X for every open set $A in Y$

Theorem (2.1) [10]
Let $f: X \longrightarrow Y$ function of a space X in to a space Ythen :-

i. $f$is a continuous function if and only if $f^{-1}(A)$ closed set in$X$.
ii. $f$is a continuous function if and only if $f(\bar{A}) \subseteq \overline{f(A)}$for every set $A in Y$.
iii. f is a continuous function if and only if $\overline{f^{-1}(A)} \subseteq f^{-1}(\bar{A})$for every set $A in Y$.
iv. f is a continuous function if and only if$f^{-1}(A^{\circ}) \subseteq \left(f^{-1}(A)\right)^{\circ}$ for every set $A in Y$.

Definition (2.2)
Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y. $f$is called s-coc-continuous function if $f^{-1}(A)$ is s-coc-open set in X for every open set A in Y .

Definition (2.3) [5]
Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y. $f$is called coc-continuous function if $f^{-1}(A)$ is coc-open set in X for every open set A in Y .

Proposition (2.1)

i. Every continuous is s-coc-continuous
ii. Every coc-continuous is s-coc-continuous

Proof

i. Let $f: X \longrightarrow Y$ be continuous function and A open set in Y. Then $f^{-1}(A)$ is open set in X . Then $f^{-1}(A)$ is s-coc-open set in X .Then f is s-coc-continuous

ii. Let $f: X \longrightarrow Y$coc-continuous function and A open set in Y then $f^{-1}(A)$ coc-open set in X . Then $f^{-1}(A)$ s-coc-open set in X then f is s-coc-continuous .

But the convers of i. and ii. not hold for example

Example (2.1)

i. Let $X = \{1,2,3,4, \dots\}$, $\tau = \{\emptyset, X, \{2\}\}$topology on X , $Y = \{a, b, c\}$, $\acute{\tau} = \{\emptyset, Y, \{a\}\}$ topology on and $f: X \longrightarrow Y defined by f(x) = \begin{cases} a if x \in \{1,3\} \\ b if x \notin \{1,3\} \end{cases}$. Then f not continuous but s-coc-continuous

ii. Let $X = \{1,2,3,4,\dots\}$, $\tau = \{\emptyset, X, \{2\}\}$topology on X , $Y = \{a,b\}, \acute{\tau} = \{\emptyset, Y, \{a\}\}$ topology on Y and $f: X \longrightarrow Y defined\ by f(x) = \begin{cases} a if x = 1 \\ b if x \neq 1 \end{cases}$. Since{a}is open set in Y and f^{-1}({a})= {1} not coc-open set in X . Then f is not coc-continuous function .But {1} s-coc-open set in X . Then f s-coc-continuous .

Proposition (2.2)

Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y then

i. The constant function is s-coc-continuous
ii. If (X,τ) discrete topology then f s-coc-continuous
iii. If X finite set and τ any topology on X then f s-coc-continuous
iv. If (Y,τ^*) indiscrete topology then f s-coc-continuous
v. The identity function is s-coc-continuous
vi. If $f: (R, U) \longrightarrow (R, \tau)$such that Uusual topology on $R and \tau$coffinite topology such that $f(x) = |x|$ for all $x \in R$ then fs-coc-continuous.

Proof clear

Proposition (2.3)
Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y then the following statements are equivalent :-

1. f s-coc-continuous function .
2. $f^{-1}(A^\circ) \subseteq \left(f^{-1}(A)\right)^{\circ s-coc}$for every set A of Y .
3. $f^{-1}(A)$s-coc-closed set in X for every closed set $A\ in\ Y$.
4. $f(\bar{A}^{s-coc}) \subseteq \overline{f(A)}$ for every set A of X .
5. $\overline{f^{-1}(A)}^{s-coc} \subseteq f^{-1}(\bar{A})$ for every set $A\ of Y$.

proof:
$1 \longrightarrow 2$
Let$A \subseteq Y$,sinceA° open set in Y .Then $f^{-1}(A^\circ)$ s-coc-open set X.Thus $f^{-1}(A^\circ) = \left(f^{-1}(A^\circ)\right)^{\circ s-coc} \subseteq \left(f^{-1}(A)\right)^{\circ s-coc}$ .Then $f^{-1}(A^\circ) \subseteq \left(f^\circ(A)\right)^{\circ s-coc}$.

$2 \longrightarrow 3$

Let $A \subseteq Y$ such that A closed set in Y.Then A^c open set then $A^c = (A^c)^\circ$ then $f^{-1}\ ((A^c)^\circ \subseteq \left(f^{-1}(A^c)\right)^{\circ s-coc}$.Then$f^{-1}(A^c) \subseteq \left(f^{-1}(A^c)\right)^{\circ s-coc}$then$\left(f^{-1}(A)\right)^c \subseteq (f^{-1}(A)^c)^{\circ s-coc}$. Therefore$\left(f^{-1}(A)\right)^c = (f^{-1}(A)^c)^{\circ s-coc}$. Hence $\left(f^{-1}(A)\right)^c$ s-coc-open set in X. Then $f^{-1}(A)$ s-coc-closed set in X.

$3 \longrightarrow 4$

Let $A \subseteq X$. Then $\overline{f(A)}$ s-coc-closed set in Y .Then by (3) we have $f^{-1}\left(\overline{f(A)}\right)$ s-coc-closed set in X containing A,thus$\bar{A}^{s-coc} \subseteq f^{-1}\left(\overline{f(A)}\right)$(since $\bar{A}^{s-coc}$ intersection of all s-coc-closed set in X containing) . Hence $f\ (\overline{(A)}^{s-coc}) \subseteq \overline{f(A)}$

$4 \longrightarrow 5$

Let $A \subseteq X$.Then$f^{-1}(A) \subseteq X$.Then by (4) we have $f\ (\overline{(f^{-1}(A)\ )}^{s-coc}) \subseteq \overline{f(f^{-1}(A))}$. Hence $\overline{f^{-1}(A)}^{\ s-coc} \subseteq f^{-1}(\bar{A})$.

$5 \longrightarrow 1$

Let B open set in Y then B^c closed .Then $B^c = \overline{B^c}$. Hence $\overline{f^{-1}(B^c)}^{\,s-coc} \subseteq f^{-1}(\overline{B^c})$. Then $\overline{f^{-1}(B^c)}^{\,s-coc} \subseteq f^{-1}(B^c)$. Then $f^{-1}(B^c) = (f^{-1}(B))^c$ s-coc-closed set in X .Therefore $f^{-1}(B)$ s-coc-open set in X . Thus f s-coc-continuous function .

Remark(2.1)

From proposition (2.1) we have f s-coc-continuous if and only if the inverseimage of every closed set in Y is s-coc- closed set in X .

Definition (2.4)

Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y then f is called s-coc irresolute ($s\text{–}co\acute{c}\text{–}continuous$) function if $f^{-1}(A)$ s-coc-open set in X for every s-coc-open set inY .

Definition (2.5)[5]

Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y then f is called coc-irresolute ($co\acute{c}\text{–}continuous$) function if $f^{-1}(A)$ coc-open set in X for every coc-open set inY .

Proposition (2.4)

Every $s\text{–}co\acute{c}\text{–}continuous$ function is s-coc-continuous function

Proof:

Let $f: (X, \tau) \longrightarrow (Y, \grave{\tau})$ $s\text{–}co\acute{c}-continuous$ and B open set in Y. Then B is s-coc-open set .Since $f s\text{–}co\acute{c}\text{–}continuous$then $f^{-1}(B)$ s-coc-open. Hence f s-coc-continuous function . But the invers is not true in general for the following example

Example(2.2)

Let $f: R \longrightarrow Y$ function defined by $f(x) = \begin{cases} 1 & if x < 0 \\ 2 & if x = 0 \\ 3 & if x > 0 \end{cases}$ and U usual topology on R , τ indiscrete topology on $Y = \{1,2,3\}$ then the only open sets in $Y$are Y$and\emptyset$, then $f^{-1}(\emptyset) = \emptyset$ $f^{-1}(Y) = \{f^{-1}(1), f^{-1}(2), f^{-1}(3)\} = \{(-\infty, 0), \{0\}, (0, \infty)\} = R$ .Since $Rand\emptyset$ s-coc-open sets in R then $f^{-1}(\emptyset), f^{-1}(Y)$ s-coc-open in R .Then f s-coc-continuous function. But $\{2\}$ s-coc-open in Y and $f^{-1}(\{2\}) = \{0\}$ is not s-coc-open set in R , $0 \in \{0\}$. There is no s-open set U such that $0 \in U$, and K compact such that $0 \in U - K \subseteq \{0\}$. Then f is not s-coc′-continuous function .

Proposition (2.5)

Let $f: X \longrightarrow Y s\text{–}co\acute{c}\text{–}continuous$ then $f^{-1}(A)$ s-coc-closed set in X for all A s-coc-closed set in Y

proof

Then A^c s-coc-open in Y .Since $f s\text{–}co\acute{c}\text{–}continuous$. Then $f^{-1}(A^c)$ is s-coc-open in X by definition (2.4) .Since $f^{-1}(A^c) = \left(f^{-1}(A)\right)^c$.Then $\left(f^{-1}(A)\right)^c$ s-coc-open set in X .Therefore $f^{-1}(A)$ s-coc-closed set in X for all A s-coc-closed set in Y.

Note that

i. $s-co\acute{c}-continuous$is need not to be continuous function .

ii. continuous is need not to be $s-co\acute{c}-$ continuous function .

iii.$s-co\acute{c}-continuous$is need not to be $co\acute{c}$–continuous function .

Examples (2.3)

i. Let $f: X \longrightarrow Y$, $X = \{1,2,3\}$, $\tau = \{\emptyset, X, \{3\}\} topology on X$ and $Y = \{a, b\}$, $\grave{\tau} = \{\emptyset, Y, \{a\}\} topology on Y$, $f(1) = f(3) = b$, $f(2) = a$. It is clear that f is s-coć-continuous but not continuous .

ii. Let $f: (Z, \tau) \longrightarrow (Y, \acute{\tau})$, $Y = \{a, b, c\}$, $\acute{\tau} = \{\emptyset, Y\}$ topology on Y ,
$\tau = \{\emptyset, Z\} topology on$ Z and $f(x) = \begin{cases} a & \text{if } x \in Z^+ \\ b & \text{if } x \in Z^- \\ c & \text{if } x = 0 \end{cases}$. Then $f$is continuous but not s-coć-continuous .

iii. Let X= $\{x_1, x_2, x_3, \ldots\}$ and $Y = \{y_1, y_2, y_3, \ldots\}$, $\tau = \{\emptyset, X, \{x_2\}\}$be topology on X , $\tau' = \{\emptyset, Y, \{y_1\}\}$ be $topology$ on Y, let $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ $defined\ by\ f(x_i) = y_i$When i= 1,2,3, ... Since $\{y_1\}$ coc-open set in Y and $f^{-1}(\{y_1\}) = \{x_1\}$ is not coc-open set in X then$f$ is not coc′-continuous . But τ'^{s-coc} is discrete topology on Y and τ^{s-coc} is discrete topology on X then$f$ is s-coc′-continuous .

Proposition (2.6)

Let $f: X \longrightarrow Y$ be a function of space X into space Y then the following statements are equivalent .

i. $f is$s-coć-continuous

ii. $f(\bar{A}^{s-coc}) \subseteq \overline{f(A)}^{s-coc}$ for every set $A \subseteq X$

iii. $\overline{f^{-1}(B)}^{s-coc} \subseteq f^{-1}(\bar{B}^{s-coc})$ for every set $B \subseteq Y$

proof:

$$i \longrightarrow ii$$

Let$A \subseteq X$then$f(A) \subseteq Y$and$\overline{f(A)}^{s-coc}$s-coc-closed set in Y .Since $f is s-co\acute{c}-continuous$. Then $f^{-1}(\overline{f(A)}^{s-coc})$s-coc-closed set in X by proposition(2.1.5) . Since$f(A) \subseteq \overline{f(A)}^{s-coc}$then$f^{-1}(f(A)) \subseteq f^{-1}(\overline{f(A)}^{s-coc})$ .Then$A \subseteq f^{-1}(\overline{f(A)}^{s-coc})$.Since $f^{-1}(\overline{f(A)}^{s-coc})$ s-coc-closed . Then$\bar{A}^{s-coc} \subseteq f^{-1}(\overline{f(A)}^{s-coc})$ .Then $f(\bar{A}^{s-coc}) \subseteq f(f^{-1}(\overline{f(A)}^{s-coc})) \subseteq \overline{f(A)}^{s-coc}$.Then $f(\bar{A}^{s-coc}) \subseteq \overline{f(A)}^{s-coc}$

$$ii \longrightarrow iii$$

Let$(\bar{A}^{s-coc}) \subseteq \overline{f(A)}^{s-coc}$ $\forall A \subseteq X$, $B \subseteq Y$,then$f^{-1}(B) \subseteq X$, $f(\overline{f^{-1}(B)}^{s-coc}) \subseteq \overline{f(f^{-1}(B))}^{s-coc}$. Since $f(f^{-1}(B)) \subseteq B$. Then $f(\overline{f^{-1}(B)}^{s-coc}) \subseteq \bar{B}^{s-coc}$ .Hence $\overline{f^{-1}(B)}^{s-coc} \subseteq f^{-1}(\bar{B}^{s-coc})$

$$iii \longrightarrow i$$

Let B s-coc-closed set in Y then B $= \bar{B}^{s-coc}$. Since $\overline{f^{-1}(B)}^{s-coc} \subseteq f^{-1}(\bar{B}^{s-coc})$. Then$\overline{f^{-1}(B)}^{s-coc} \subseteq f^{-1}(B)$.Since $f^{-1}(B) \subseteq \overline{f^{-1}(B)}^{s-coc}$.Therefore $\overline{f^{-1}(B)}^{s-coc} = f^{-1}(B)$. Therefore $f^{-1}(B)$ s-coc-closed in X .Then $iss - co\acute{c} - continuous$.

Not that

Acomposition of two s-coc-continuous function is not necessary be s-coc-continuous function as the following example.

Example (2.4)

Let $X = R$, $Y = \{1,2,3\}$, $W = \{a, b\}$, $\tau = \{\emptyset, R, R^-\}$topology on R , $\acute{\tau} = \{\emptyset, Y, \{2,3\}\}$ topology on Y , $\tau^* = \{\emptyset, W, \{a\}\}$ be topology on W and $f: R \longrightarrow Y$ defined by $f(x) = \begin{cases} 1 & \text{if } x = 0 \\ 2 & \text{if } x \in R^+ \\ 3 & \text{if } x \in R^- \end{cases}$, $g: Y \longrightarrow Z$ defined by $g(x) = \begin{cases} a \ if \ x \in \{1,2\} \\ b \ if \ x = 3 \end{cases}$are s-coc-continuous in Y , W , but $g \circ f$ not s-coc-continuous , since $(g \circ f)(\{a\}) = f^{-1}(\{1,2\}) = \{0, R^+\}$ not s-coc-open .Then $g \circ f$ not s-coc-continuous .

Proposition (2.7)

If $f: X \longrightarrow Y$ s-coc-continuous and $g: Y \longrightarrow W$ continuous then $g \circ f$ s-coc-continuous

proof:

Let B open set in W .Since g continuous , then $g^{-1}(B)$ open in Y .Since f s-coc-continuous then $f^{-1}\left(g^{-1}(B)\right)$ s-coc-continuous in X .Then $(g \circ f)^{-1}(B)$ s-coc-open in X.Then $g \circ f: X \longrightarrow Z$ s-coc-continuous

Proposition (2.8)

If $f: X \longrightarrow Y$ and $g: Y \longrightarrow W s\text{–} co\acute{c}\text{–} continuous$ then $g \circ f \ is \ s\text{–} co\acute{c}\text{–} continous$.

Proposition (2.9)

Let $f: X \longrightarrow Y$ be a function of space X into space Y then f is $s\text{–} co\acute{c}\text{–} continuous$ function if and only if the inverse image of every s-coc-closed in Y is s-coc-closed set in X

proof:

Let $s\text{–} co\acute{c}\text{–} continuous$, let B s-coc-closed set in Y .Then B^c s-coc-open in Y Since $s\text{–} co\acute{c}\text{–} continuous$, then $f^{-1}(B^c)$ s-coc-open in X . $f^{-1}(B^c) = f^{-1}(B - Y) = f^{-1}(Y) - f^{-1}(B) = X - f^{-1}(B) = (f^{-1}(B))^{\ c}$ .Then $(f^{-1}(B))^{\ c}$ s-coc-open in X. Hence $f^{-1}(B)$ s-coc-closed in X

Conversely:

Let M s-coc-open in Y, thenM^cs-coc-closed in Y . Then $f^{-1}(\ M^c\ )$ s-coc-closed in X, since $f^{-1}(M^c) = \mathrm{f}^{-1}(\mathrm{Y} - \mathrm{M}) = \mathrm{f}^{-1}(\mathrm{Y}) - \mathrm{f}^{-1}(\mathrm{M}) = \mathrm{X} - \mathrm{f}^{-1}(\mathrm{M}) = (\mathrm{f}^{-1}(\mathrm{M}))^{\ \mathrm{c}}$.

Therefore$\mathrm{f}^{-1}(\mathrm{M}^\mathrm{c}) = (\mathrm{f}^{-1}(\mathrm{M}))^{\ \mathrm{c}}$. Then $(\mathrm{f}^{-1}(\mathrm{M}))^{\ \mathrm{c}}$ s-coc-closed in X hence $\mathrm{f}^{-1}(\mathrm{M})$ s-coc-closed in X $t$hen f is $\mathrm{s} - \mathrm{co\acute{c}} - \mathrm{continuous}$.

Proposition (2.10)

If $f: X \longrightarrow Y$ s-coc-continuous onto then for all $y \in Y$ and for all U nbd of y we get there exists A s-coc-open of $f^{-1}(y)$such that $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ s-coc-nbd of $f^{-1}(y)$

Proof :

Let $y \in Y$ and U nbd of y . Then there exists B open set in Y such that $y \in B \subseteq U$ .Since f onto then there exists $x \in X$ such that $(x) = y$. Then $x = f^{-1}(y)$. Since f s-coc-continuous then $f^{-1}(B)$ s-coc-open in X, $f^{-1}(y) \in f^{-1}(B) \subseteq f^{-1}(U)$. Let $A = f^{-1}(B)$ then $f^{-1}(y) \in A \subseteq f^{-1}(U)$ then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ s-coc-nbd of $f^{-1}(y)$

Proposition (2.11)

If A is locally s-coc-closed set in Y and $f:(X,\tau) \longrightarrow (Y,\acute{\tau})$ continuous and s-coć-continuous . Then f^{-1}(A) is locally s-coc-closed set in X.

Proof :

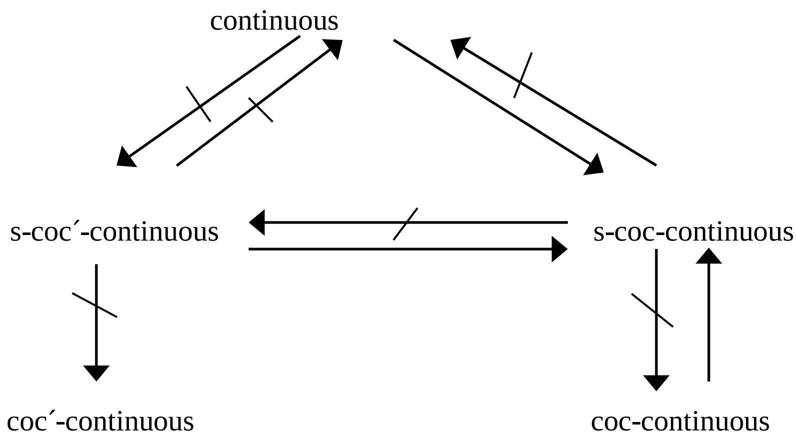
Since A is locally s-coc-closed set in Y .Then $A = U \cap F$such that $U \in \acute{\tau}$ and F s-coc-closed set in Y**.** Since f is continuous. Then $f^{-1}(U)$ open set in X **.**Since f s-coć-continuous. Then $f^{-1}(F)$ is s-coc-closed set in $X$and $f^{-1}(A) = f^{-1}(U \cap F) = f^{-1}(U) \cap f^{-1}(F)$then f^{-1}(A) is locally s-coc-closed set in X.

Proposition (2.12)

If A is locally s-coc-closed set in Y and $f:(X,\tau) \longrightarrow (Y,\acute{\tau})$ continuous. Then f^{-1}(A) is locally s-coc-closed set in X

Proof :

Since A is locally s-coc-closed set in Y.Then $A = U \cap F$ such that $U \in \acute{\tau}$ and F s-coc-closed set in Y . Since f is continuous. Then $f^{-1}(U) \in \tau$ and $f^{-1}(F)$ is closed sets in X .Then$f^{-1}(F)$ is s-coc-closed set in X .Since $f^{-1}(A) = f^{-1}(U \cap F) = f^{-1}(U) \cap f^{-1}(F)$ Then f^{-1}(A) is locally s-coc-closed set in X



3.On s-coc-separation axioms

In this section we recall some definitions , examples , remarks and propositions about separation properties . by using s-coc-open sets and we prove some relation between them .

Definition (3.1) [3]

A space is called T_1-space if and only if for each $x \neq y \text{ in } X$ there exists open sets $U \text{ and } V$ such that $x \in U$, $y \notin U \text{ and } y \in V$, $x \notin V$ **.**

Definition (3.2)

A space is called s-coc-T_1-space if and only if for each $x \neq y \; in \; X$ there exists s-coc-open sets $U \; and \; V$ such that $x \in U$,$y \notin U \; and \; y \in V$, $x \notin V$ **.**

Remark (3.1)

Every T_1-space is s-coc-T_1-space but the convers is not true in general .

Example (3.1)

Let $X = \{1,2,3,4, \dots\}$, $\tau = \{\emptyset, X, \{1\}, \{1,2\}\}$Topology on X . The s-coc-open sets discrete Topology then X is s-coc-T_1-space but not T_1-space.

Proposition (3.1)

Let X be a space , then X is s-coc-T_1-space if and only if $\{x\}$ s-coc-closed set for each $x \in X$.

Proof

Let X is s-coc-T_1-space and $y \in X$ such that$y \notin \{x\}$. Then $y \neq x$, since X is s-coc-T_1-space , then there exists s-coc-open set V such that $y \in V$, $y \notin \{x\} \; and \; x \notin V$, $x \in \{x\}$ .Then $V \cap \{x\} = \emptyset \;\; then \;\; (V - y) \cap \{x\} = \emptyset$,then $y \notin \{x\}'^{s-coc}$ hence $\{x\}'^{s-coc} \subseteq \{x\}$.Then $\{x\}$ s-coc-closed by proposition (1.1.14) (2) .

Conversely :

Let $\{x\}$ s-coc-closed $\forall \; x \in X$ then $\{x\}^c$ s-coc-open set ,let$x \neq y \; in \; X$ then $y \in \{x\}^c$, $x \notin \{x\}^c$. Then $\{x\}^c = X - \{x\}$since $\{y\}$ s-coc-closed then $\{y\}^c$ s-coc-open $\{y\}^c = X - \{y\} \; and \; y \notin \{y\}^c$, $x \in \{y\}^c$. Hence X is s-coc-T_1-space .

Definition (3.3) [1]

Let $f: X \longrightarrow Y$ be a function of space X into space Y then :-

i- $f$is called open function if $f(A)$ is open set in Y for every open set $A \; in \; X$.

ii- f is called closed function if $f(A)$ is closed set in Y for every closed set $A \; in \; X$.

Definition (3.4)

A function $f: (X, \tau) \longrightarrow (Y, \acute{\tau})$ is called

i- super s-coc-open if $f(U)$ is open in Y for each U s-coc-open in X.

ii- super s-coc-closed if $f(U)$ is closed in Y for each U s-coc-closed in X.

Proposition (3.2)

If X is s-coc-T_1-space and $f: X \longrightarrow Y$ super s-coc-open bijective then$Y$ is T_1-space .

Proof

Let $x, y \in Y$ such that $x \neq y$ since f onto then there exist $a, b \in X$ such that $f(a) = x$, $f(b) = y$ Then $a \neq b$ since X is s-coc-T_1 .Then there exists U, V s-coc-open sets such that ($a \in U, b \notin U$) and ($a \notin V, b \in V$).Since f super s-coc-open then $f(U)$, $f(V)$ open in Y.Then $\big(f(a) \in f(U) \, , f(b) \notin f(U)\big)$ and $\big(f(a) \notin f(V) \, , f(b) \in f(V)\big)$ .Thus Y is T_1-space .

Definition (3.5) [5]

Let $f: X \longrightarrow Y$ be a function of space X into space Y then :

i- f is called coc-closed function if $f(A)$ is coc-closed set in Y for every closed set $A \; in \; X$.

ii- f is called coc-open function if $f(A)$ is coc-open set in Y for every open set $A \; in \; X$.

Proposition (3.3)

Let $f: X \longrightarrow Y$ onto s-coc-open function. If X is T_1-space then Y is s-coc-T_1-space .

Proof

Let $y_1, y_2 \in Y \ni y_1 \neq y_2$. Since $f: X \longrightarrow Y$ onto function, then there exists $x_1, x_2 \in X$ such that $(x_1) = y_1$, $f(x_2) = y_2$, Then $x_1 \neq x_2$. Since X is T_1-space then there exists U , V open sets in X such that $(x_1 \in U, x_2 \notin U)$ and $(x_2 \in V, x_1 \notin V)$.Since f s-coc-open function. Then $f(U)$, $f(V)$ s-coc-open sets in Y. Since $x_1 \in U$ then $f(x_1) \in f(U)$ and $x_1 \notin V$then $(x_1) \notin f(V)$. Since $x_2 \in U$ then $f(x_2) \notin f(U)$ and $x_2 \notin V$ then $f(x_2) \in f(V)$ .Then Y is s-coc-T_1-space.

Proposition (3.4)

Let $f: X \longrightarrow Y$ one-to-one s-coc-continuous function. If Y isT_1-space then X is s-coc-T_1 – space.

Proof :

Let $x_1, x_2 \in X$ $such\ that$ $x_1 \neq x_2$ .Since $f: X \longrightarrow Y$ one-to-one function and $x_1 \neq x_2$ Then $f(x_1) \neq f(x_2)$ .Since Y isT_1-space then there exists U, V open sets in Y such that $(f(x_1) \in U$, $f(x_2) \notin U)$and $(f(x_2) \in V$, $f(x_1) \notin V)$. Since f s-coc-continues function then $f^{-1}(U), f^{-1}(V)$ s-coc-open sets in X .Since $f(x_1) \in U$ then $x_1 \in f^{-1}(U)$ and $f(x_2) \notin U$ then $x_2 \notin f^{-1}(U)$.Since $f(x_2) \in V$ then $x_2 \in f^{-1}(V)$ and $f(x_1) \notin V$ then $x_1 \notin f^{-1}(V)$hence X is s-coc-T_1-space

Definition (3.6)

Let $f: X \longrightarrow Y$ be a function of space X into space Y then :

1) f is called s-coc-closed function if $f(A)$ is s-coc-closed set in Y for every closed set $A\ in\ X$.
2) f is called s-coc-open function if $f(A)$ is s-coc-open set in Y for every open set $A\ in\ X$.

Definition (3.7)[5]

Let $f: X \longrightarrow Y$ be a function of space X into space Ythen :-

i. f is called co$\acute{c}$-closed function if $f(A)$ is coc-closed set in Y for all coc-closed A in X.
ii. $f i$ s called co$\acute{c}$-open function if $f(A)$ is coc-open set in Y for all co$c$-open A in X.

Definition (3.8)

Let $f: X \longrightarrow Y$ be a function of space X into space Ythen :-

1) f is called s-co$\acute{c}$-closed function if $f(A)$ is s-coc-closed set in Y for all s-coc-closed set A in X.
2) f is called s-co$\acute{c}$-open function if $f(A)$ is s-coc-open set in Y for all s-co$c$-open set A in X.

Definition (3.9)

Let X and Y are spaces. Then a function $f: X \longrightarrow Y$ is called s-coc-homeomorphism if

1. f bijective
2. f s-coc-continuous
3. f s-coc-closed (s-coc-open)

It is clear that every homeomorphism is s-coc-homeomorphism .

Definition (3.10)

Let X and Y are spaces ,then a function $f: X \longrightarrow Y$ is called s-co$\acute{c}$-homeomorphism if :-

1. f bijective
2. f s-co$\acute{c}$-continuous
3. f s-co$\acute{c}$-closed (s-coc-open)

It is clear that every homeomorphism is s-co$\acute{c}$-homeomorphism

Theorem (3.1)

Let X and Y be s-coć-homeomorphism space then X s-coc-T$_1$-space iff Y is s-coc-T$_1$-space.

Proof: Clear

Definition (3.11) [3]

A space is called T$_2$-space (Hausdorff) if and only if for each $x \neq y \; in \; X$ there exists disjoint open sets $U \; and \; V$ such that $x \in U$, $y \in V$.

Definition (3.12)

A space is called s-coc-T$_2$-space (s-coc-Hausdorff) if and only if for each $x \neq y \; in \; X$ there exists $U \; and \; V$ disjoint s-coc-open sets such that $x \in U$, $y \in V$.

Remark (3.2)

It is clear that every T$_2$-space is s-coc-T$_2$-space but the converse is not true for example

Example (3.2)

Let $X = \{1,2,3,4, \dots\}$, $\tau = \{\emptyset, X, \{1\}\}$Topology on X.τ^{sk}discrete Topology. Then X is s-coc-T$_2$-space but not T$_2$-space.

Proposition (3.5)

Let $f: X \longrightarrow Y$ be bijective s-coc-open function . If X is T$_2$-space then Y is s-coc-T$_2$-space.

Proof :

Let$y_1, y_2 \in Y \ni y_1 \neq y_2$.Since$f: X \longrightarrow Y$bijective function,then $there \; exists \; x_1, x_2 \in X such \; that \; f(x_1) = y_1$, $f(x_2) = y_2$, Then $x_1 \neq x_2$ since f s-coc-open function ,Since X is T$_2$-space and $x_1 \neq x_2$ then $there \; exists \; U$,V open sets in X such that $x_1 \in U$,$x_2 \in V \; and \; U \cap V = \emptyset$ .Since f s-coc-open then $f(U)$,$f(V)$ s-coc-open sets in Y . Since $x_1 \in U$ then $f(x_1) \in f(U)$ and $x_1 \notin V$ then $f(x_1) \notin f(V)$ and $f(U) \cap f(V) = f(U \cap V) = \emptyset$ .Hence Y is s-coc-T$_2$-space

Remark (3.3)

Every is s-coc-T$_2$ space is s-coc-T$_1$space . But the converse is not true for the following example .

Example (3.3)

Let $X = R \; and \; \tau$ coffinite Topology on R **t**hen τ^{sk} coffinite Topology **,** let $x \neq y$ **.**Since $x \notin R - \{x\}$, $x \in R - \{y\}$ and $y \in R - \{x\}$, $y \notin R - \{y\}$.Since $R - \{x\}$, $R - \{y\}$open sets then for all $\neq y \; in \; R$, $R - \{x\}$, $R - \{y\}$ are s-coc-open sets .Then R is s-coc-T$_1$-space . Since $3 \neq 4$ and there is no disjoint s-coc-open sets U, V such that $3 \in U, 4 \in V$ .Thus **(**R, τ) is not s-coc-T$_2$-space .

Proposition (3.6)

If X is s-coc-T$_2$-space and $f: X \longrightarrow Y$ super s-coc-open bijective then Y is T$_2$-space .

Proof

Let $x, y \in Y$ such that $x \neq y$. Since f onto then there exist $a, b \in X$ such that $f(a) = x$, $f(b) = y$ and a $\neq b$. Since X is s-coc-T$_2$. Then there exists U, V s-coc-open sets such that a$\in U, b \in V$ .Since f super s-coc-open then $f(U)$, $f(V)$ open sets in Y. Since $U \cap V = \emptyset$ then $f(U) \cap f(V) = f(U \cap V) = \emptyset$.Thus$f(U)$, $f(V)$disjoint sets .Then $f(a) \in f(U) \; and \; f(b) \in f(V)$ .Then for all $f(a) = x$, $f(b) = y \in Y$ such that $x \neq y$ .There exists disjoint open sets in Y such that $x \in f(U), y \in f(V)$ .Thus Y is T$_2$-space .

Proposition (3.7)

Let $f: X \longrightarrow Y$ one-to-one s-coc-continuous function. If Y isT_2-space then X is s-coc-T_2-space.

Proof

Let $x_1, x_2 \in X \; such\ that \; x_1 \neq x_2$. Since $f: X \longrightarrow Y$ one-to-one function and $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ in Y . Since Y isT_2-space then $\exists\, U, V$ open sets in $suchthat\ (x_1 \in U, x_2 \in V)\ and\ U \cap V = \emptyset$. Since f s-coc-continuous function then $f^{-1}(U), f^{-1}(V)$ s-coc-open sets in X , since $f(x_1) \in U$ then $x_1 \in f^{-1}(U)$. Since $f(x_2) \in V$ then $x_2 \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = \emptyset$ hence X is s-coc-T_2-space .

Theorem (3.2)

Let X and Y be s-coć-homeomorphism space then X s-coc-T_2-space if and only if Y is s-coc-T_2-space.

Proof

Let$X$,Y be s-coc´-homeomorphism ,let$X$ s-coc-T_2-space .Let $y_1, y_2 \in Y\ suchthat\ y_1 \neq y_2$.Since f onto function .Then $there\ exist\ x_1, x_2 \in X\ suchthat\ f(x_1) = y_1, f(x_2) = y_2$ since X is s-coc-T_2-space ,Then $there\ exists\ U, V$ s-coc-open sets in $X\ suchthat(x_1 \in U, x_2 \in V)\ and\ U \cap V = \emptyset$. Since f s-coć-open function .Then $f(U), f(V)$ s-coc-open sets in Y.Since $x_1 \in U$ then $f(x_1) \in f(U)$ and $x_2 \in V$ then $f(x_2) \in f(U)$then $f(U) \cap f(V) = f(U \cap V) = \emptyset$ Then Y s-coc-T_2-space .

Conversely

Let $x_1, x_2 \in X\ suchthat\ x_1 \neq x_2$ .Since f one-to-one function and $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ .Since Y is T_2-space $then\ there\ exists\ U, V$ s-coc open sets in $Y\ suchthat\ (x_1 \in U, x_2 \in V) and\ U \cap V = \emptyset$. Since f s-coc-continuous function then $f^{-1}(U), f^{-1}(V)$ s-coc-open sets in X .Since $f(x_1) \in U$then $x_1 \in f^{-1}(U)$. Since $f(x_2) \in V$ then $x_2 \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ hence X is s-coc-T_2-space .

Definition (3.13) [8]

A space X is said to be regular space if and only if for each $x \in X$ and C closed subset such that $x \notin C$ there exist disjoint open sets U, V such that x $\in U$ and $C \subseteq V$.

Definition (3.14)

A space X is said to be s-coc-regular space if and only if for each $x \in X$ and B closed subset of X such that $x \notin B$ there exist disjoint s-coc-open sets U, Vsuch that $x \in U\ and\ B \subseteq V$.

Remark (3.4)

Every regular space is s-coc-regular but the convers is not true .

Example (3.4)

Let X $= \{1,2,3\}, \tau = \{\emptyset, X, \{2,3\}\}$. X s-coc-regular (Since s-coc-open sets is discrete)and$F = \{1\}$closed set and $2 \notin \{1\} = F there\ exists\ no$ open sets $U, V\ suchthat\ 2 \in U = \{2,3\}, F \subseteq V = X\ and\ U \cap V \neq \emptyset$ .Then X is not regular .

Proposition (3.8)

If f: X $\longrightarrow$ Y homeomorphism and s-coć- homeomorphism. Then X is s-coc-regular if and only if Y is s-coc-regular

Proof

Let Y is s-coc-regular. To prove X s-coc-regular , let $x \in X$ and F closed set in X such that $x \notin F$. Then $f(x) \notin f(F)$ since f is (closed) function then f (F) closed set inY. Since Y is s-coc-regular then there exists U, V disjoint s-coc-open sets such that $f(x) \in U, f(F) \subseteq V$.

Since f s-coć-continuous then $f^{-1}(U), f^{-1}(V)$ s-coc-open sets in X.Since $f^{-1}(\emptyset) = f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$. Then $f^{-1}(U), f^{-1}(V)$ disjoint sets in X .Then $f^{-1}\big(f(x)\big) \in f^{-1}(U)$, $f^{-1}\big(f(F)\big) \subseteq f^{-1}(V)$. Since f bijective then $x \in f^{-1}(U)$, $F \subseteq f^{-1}(V)$. Then X is s-coc-regular.

Conversely:

Let X s-coc-regular . To prove Y is s-coc-regular ,let $y \in Y$ and F closed set in Y such that $y \notin F$ .Since f onto then there exists $x \in X$ such that $f(x) = y$ .Then $x = f^{-1}(y)$.Since f continuous then $f^{-1}(F)$ closed set in X and $x \notin f^{-1}(F)$ .Since X s-coc-regular then there exists U, V disjoint s-coc-open sets such that $x \in U, f^{-1}(F) \subseteq V$. Since f s-coć-continuous then $f(U), f(V)$ s-coc-open sets in Y . Since $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$.Thenf$(U), f(V)$ disjoint sets in Xand $y = f(x) \in f(U), f\big(f^{-1}(F)\big) \subseteq f(V)$ .Since f bijective then $y \in f(U)$, $F \subseteq f(V)$.Then Y is s-coc-regular.

Proposition (3.9)

A space X is s-coc-regular space if and only if for every $x \in X$ and every open set $U \; in \; X$ such that $x \in U$ there exist s-coc-open set W such that $x \in W \subseteq \overline{W}^{s-coc} \subseteq U$.

Proof

Let X s-coc-regular space and $x \in X$, U open set in X such that $x \in U$, Then U^c closed set in X and $x \notin U^c$ then there exist disjoint s-coc-open sets W, V such that $x \in W, U^c \subseteq V$. Hence $x \in W \subseteq \overline{W}^{s-coc} \subseteq \overline{V^c}^{s-coc} \subseteq V^c \subseteq U$.

Conversely

Let $x \in X \; and \; F$ closed set in X such that $x \notin F$ then F^c open set in X and $x \in F^c$, then there exist s-coc-open sets W such that $x \in W \subseteq \overline{W}^{s-coc} \subseteq F^c$. Then $x \in W, F \subseteq \left(\overline{W}^{s-coc}\right)^c$ are disjoint s-coc-open sets .Then X s-coc-regular space

Proposition (3.10)

If $f: X \longrightarrow Y$onto , continuous , s-coc-open function and X regular space then Y s-coc-regular.

Proof

Let $y \in Y \; and \; C$ closed set such that $y \notin C$ .Since f onto then there is $x \in X$ such that $f(x) = y$. Since f continuous and C closed set in Y. Then $f^{-1}(C)$ closed in X and $x = f^{-1}(y) \notin f^{-1}(C)$. Since X regular space then there is U, V open disjoint sets such that $x \in U \; and \; f^{-1}(C) \subseteq V$ .Since f s-coc-open then $f(U), f(V)$ s-coc-open sets and disjoint.Then $y = f(x) \in f(U)$ and $C \subseteq f(V)$ .Thus Y s-coc-regular space

Proposition (3.11)

If $f: X \longrightarrow Y$ closed bijective , s-coc-continuous and if Y regularthen X s-coc-regular **.**

Proof

Let $x \in X \; and \; F$ closed set in X such that $x \notin F$ .Since f closed function then $f(F)$ closed set in Y such that $f(x) \notin f(F)$ .Since Y regular space then there is U, V open disjoint sets such that $f(F) \subseteq V \; and \; f(x) \subseteq U$ .Since$f$ s-coc-continuous then $f^{-1}(U), f^{-1}(V)$ s-coc-open sets and disjoint and $f^{-1}\big(f(F)\big) \subseteq f^{-1}(V), f^{-1}\big(f(x)\big) \subseteq f^{-1}(U)$ .Since f bijective then $f^{-1}\big(f(F)\big) = F \; and \; f^{-1}\big(f(x)\big) = x$. Since $U \cap V = \emptyset$ then $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$.Then X s-coc-regular .

Not that

i. If X is s-coc-T_1-space then X need not to be s-coc-regular .
ii. If X is s-coc-regular then X need not to be s-coc-T_1.

Example (3.5)

i. Let $X = R$ and τ coffinite Topology on R . Since $3 \in R$ and $\{1\}$ closed set such that $3 \notin \{1\}$and there is no disjoint s-coc-open sets U, V such that $3 \in U, \{1\} \in V$ then R is not s-coc-regular . But R is s-coc-T_1

ii. Let $X = \{1,2,3,4, \dots\}$ and $\tau = \{\emptyset, X\}$. Since there is no closed set C such that $x \notin C$ where $x \in X$ then X is s-coc-regular .Since $\tau^{sk} = \{\emptyset, X\}$ and $1,2 \in X$ and $1 \neq 2$. There is no U, V s-coc-open sets such that $1 \in U, 1 \notin V$ and $2 \notin U$, $2 \in V$ .Then X is not s-coc-T_1space .

Proposition (3.12)

If $f: X \longrightarrow Y$ super s-coc function, continuous , onto and X s-coc-regular then Y is regular.

Proof

Let $y \in Y$ and B closed set in Y such that $y \notin B$ .Since f continues then $f^{-1}(B)$ closed in X. Since f onto then there is $x \in X$ such that $f(x) = y$ and $x \notin f^{-1}(B)$.Since X is s-coc-regular then there exists U, V disjoint s-coc-open sets in X such that $x \in U$ and $f^{-1}(B) \subseteq V$.Since f super s-coc-open then $f(U), f(V)$ open in Y.Thus $y = f(x) \in f(U)$ and $B \subseteq f(V)$ .Then Y is regular space .

Definition (3.15) [11]

A space X is normal if and only if when every A and B are disjoint closed subsets in X , there exist disjoint open sets U, V with $A \subseteq U$ and $B \subseteq V$ **.**

Definition (3.16)

A space X is called s-coc-normal space if and only if when every disjoint closed sets C_1 , C_2 there exist disjoint s-coc-open sets V_1, V_2 such that $C_1 \subseteq V_1$ and $C_2 \subseteq V_2$.

Remark (3.5)

It is clear that every normal space is s-coc-normal space .but the converse is not true .

Example (3.6)

Let $X = \{1,2,3,4\}$, $\tau = \{\emptyset, X, \{1,2,3\}, \{1,3,4\}, \{1,3\}$.The closed sets in X are $\{\emptyset, X, \{4\}, \{2\}, \{2,4\}\}$ and τ^{sk} is discrete Topology then X is s-coc-normal .But not normal since $\{2\}, \{4\}$ disjoint closed sets and there exists no disjoint open sets $V_1, V_2 \ni \{2\} \subseteq V_1$, $\{4\} \subseteq V_2$.

Remark (3.6)

1. If X is s-coc-T_1-space then X need not to be s-coc-normal.
2. If X is s-coc-normal then X need not to be s-coc-regular.
3. If X is s-coc-normal then X need not to be s-coc-T_1.

Example (3.7)

1. Let $X = R$, τ coffinite Topology on R . Since $\{1\},\{2\}$ disjoint closed sets and there is no s-coc-open sets U, V such that $\{1\} \subseteq U, \{2\} \subseteq V$then R is not s-coc-normal But (X,τ) is s-coc-T_1 by example (3.1.3) .

2. Let $X = Z \; and \; \tau = \{\emptyset, Z, Z_\circ, Z_\circ^+, Z_\circ^-\}$. Since $Z_e$closed set and $1 \notin Z_e$. there is no disjoint s-coc-open setsU, V such that $1 \in U \; and \; Z_e \subseteq V$ then Z is not s-coc-regular. But the closed sets are $Z_e, (Z_\circ^-)^c = \{Z_e, Z_\circ^+\}, (Z_\circ^+)^c = \{Z_e, Z_\circ^-\}, \emptyset , Z_e, \; and \; Z$ are not disjoint closed sets then Z is s-coc-normal.

3. Let $X = \{1,2,3,4, \dots\}$, $\tau = \{\emptyset, X\}$ then $\tau^{sk} = \{\emptyset, X\}$ .Since there is no C_1 , C_2 dis joint closed sets then X is s-coc-normal . If each $x, y \in X \; such \; that \; x \neq y$ since there exists no s-coc-open sets U, V such that $x \in U, x \notin V and \; y \notin U, y \in V$ then X is not s-coc-T_1space .

Proposition (3.13)

If $f: X \longrightarrow Y$ homeomorphism and s-coć- homeomorphism .Then X is s-coc-normal if and only if Y s-coc-normal

Proof

To prove X s-coc-normal let Y be s-coc-normal, let A, B disjoint closed sets in X .Since f closed function, then f (A), f (B) disjoint closed sets in Y . Since Y s-coc-normal then there exists U, V disjoint s-coc-open set in Y such that f (A) ⊆ U, f (B) ⊆ V. Since f s-coć-continuous then f^{-1} (U), f^{-1} (V) disjoint s-coc-open sets in X such that f^{-1} (f (A)) ⊆ f^{-1}(U), f^{-1}(f (B)) ⊆ f^{-1}(V). Since f one to one then A ⊆ f^{-1} (U), B ⊆ f^{-1} (V) .Then X s-coc-normal .

Conversely:

Let X s-coc-normal , to prove Y s-coc-normal , let A, B disjoint closed sets in Y . Since f continuous then f^{-1} (A), f^{-1} (B) disjoint sets in X Since X s-coc-normal then there exists disjoint s-coc-open sets U, V such that f^{-1} (A) ⊆ U, f^{-1} (B) ⊆ V . Since f s-coć-open then f (U), f (V) disjoint s-coc-open sets in Y such that f(f^{-1}(A)) ⊆ f (U), f (f^{-1}(B)) ⊆ f (V) . Since f bijective then A ⊆ f (U), B ⊆ f (V) . Then Y s-coc-normal

Proposition (3.14)

If X s-coc-normal and $f: X \longrightarrow Y$ super s-coc-open, continuous and one to one then Y is normal .

Proof

Let F_1, F_2disjoint closed sets in Y.Since f continuous then $f^{-1}(F_1), f^{-1}(F_2)$ closed sets in X.Since $F_1 \cap F_2 = \emptyset$ then $f^{-1}(F_1) \cap f^{-1}(F_2) = f^{-1}(F_1 \cap F_2) = \emptyset$.Then $f^{-1}(F_1) \cap f^{-1}(F_2)$ disjoint sets in X.Since X s-coc-normal then there exist disjoint s-coc-open sets U, V such that $f^{-1}(F_1) \subseteq U \; and \; f^{-1}(F_2) \subseteq V$.Since f super s-coc-open then $f(U) \; and \; f(V)$ open sets in Y . Since $U \cap V = \emptyset$then $f(U) \cap f(V) = f(U \cap V) = \emptyset$.Since f one to one then $f\left(f^{-1}(F_1)\right) = F_1 \; and \; f\left(f^{-1}(F_2)\right) = F_2$ .Then $F_1 \subseteq f(U) \; and \; F_2 \subseteq f(V)$.Then Y is normal space .

Proposition (3.15)

Let $f: X \longrightarrow Y$ continuous and s-coc-open function if X normal then Y s-coc-normal .

Proof

Let A_1, A_2disjoint closed sets in Y then $f^{-1}(A_1), f^{-1}(A_2)$ closed disjoint sets in X. Since X normal then there is U, V open disjoint sets such that $f^{-1}(A_1) \subseteq U \; and \; f^{-1}(A_2) \subseteq V$.Since f s-coc-open then $f(U) \; and \; f(V)$ s-coc-open sets in Y and disjoint such that $A_1 \subseteq f(U) \; and \; A_2 \subseteq f(V)$then Y is s-coc-normal space .

Proposition (3.16)

Let $f: X \longrightarrow Y$ s-coc-continuous and closed function , if Y normal space then X s-coc-normal

Proof

Let A_1, A_2disjoint closed sets in X.Since f closed function then $f(A_1), f(A_2)$ closed disjoint sets in Y.Since Y normal space then there is U, V open disjoint sets such that $f(A_1) \subseteq U \, and \, f(A_2) \subseteq V$.Since f s-coc-continuous then $f^{-1}(U), f^{-1}(V)$ disjoint s-coc-open sets then $A_1 \subseteq f^{-1}(U) \, and \, A_2 \subseteq f^{-1}(V)$ Thus X is s-coc-normal space .

Proposition (3.17)

A space X is s-coc-normal space if and only if for every closed set $D \in X$ and each open set $U \, in \, X$ such that $D \subseteq U$ there exist s-coc-open set V such that $\subseteq V \subseteq \overline{V}^{s-coc} \subseteq U$.

Proof

Let X s-coc-normal and let $D \in X$ closed set and U open set in X such that $D \subseteq U$ then D, U^c disjoint closed sets .Since X is s-coc-normal space then there exist disjoint s-coc-open sets V, W such that $D \subseteq V, U^c \subseteq W$ then $D \subseteq V \subseteq \overline{V}^{s-coc} \subseteq \overline{W^c}^{s-coc} \subseteq W^c \subseteq U$.Then $D \subseteq V \subseteq \overline{V}^{s-coc} \subseteq U$.

Conversely:

Let D_1, D_2disjoint closed sets in X then ${D_2}^c$ open set in X and $D_1 \subseteq {D_2}^c$.Then there exists s-coc-open set V such that $D_1 \subseteq V \subseteq \overline{V}^{s-coc} \subseteq {D_2}^c$ then $D_1 \subseteq V, D_2 \subseteq \overline{V}^{s-coc}$.Since V, $\left(\overline{V}^{s-coc}\right)^c$ are disjoint s-coc-open sets hence X s-coc-normal space .

Proposition (3.18)

If X s-coc-normal space and T_1-space, then X s-coc-regular space .

Proof

Let $x \in X \, and \, W$ closed set in X such that $x \in W$ .Then $\{x\}$ closed subset of X .Since X is T_1-space then $\{x\} \subseteq W$ .Since X s-coc-normal space. Thus there exists s-coc-open set V such that $\{x\} \subseteq V \subseteq \overline{V}^{s-coc} \subseteq W$ by proposition (3.17). So that $x \in V \subseteq \overline{V}^{s-coc} \subseteq W$.Therefore X s-coc-regular space by proposition (3.9) .

Definition (3.17)

A space X is said to be s-coc*-regular if for all $x \in X$ and all F s-coc-closed set such that $x \notin F$ there exist two disjoint s-coc-open sets U, V such that $x \in U, F \subseteq V$.

Proposition (3.19)

Every s-coc*-regular is s-coc-regular

Proof

Let F is closed set in Xand $x \notin F$. Then F is s-coc-closed set, since $X$is s-coc*-regular .Then there exist two disjoint s-coc-regular sets U, V such that $x \in U, F \subseteq V$. Then Xs-coc-regular .

Proposition (3.20)

If $f: X \longrightarrow Y$ onto super s-coc-open continuous and X is s-coc^*-regular then Y is regular

Proof

Let $y \in Y$, F closed set in Y such that $y \notin F$. Since f continuous then $f^{-1}(F)$ closed in X .Since f onto then there exists $x \in X$ such that $f(x) = y$ .Then $x = f^{-1}(y)$ and $x \notin f^{-1}(F)$ .Since X s-coc^*-regular then there exist U, V disjoint s-coc-open sets such that $x \in U, f^{-1}(F) \subseteq V$ .Since f is super s-coc-open then $f(U), f(V)$ disjoint open sets in Y, and $f(x) \in f(U), F \subseteq f(V)$Then Y is regular

Proposition (3.21)

Let X s-coc^{*}-regular and $f: X \longrightarrow Y$ onto, s-coc-continuous and s-co$\acute{c}$-open. Then Y is s-coc-regular.

Proof

Let $y \in Y, F$ closed set in Y such that $y \notin F$ .Since f onto then there exists $x \in X$ such that $f(x) = y$ .Then $x = f^{-1}(y)$ .Since f continuous then $f^{-1}(F)$ closed set in X and $x \notin f^{-1}(F)$ .Since X s-coc^{*}-regular then there exist two disjoint s-coc-open sets U, V such that $x \in U, f^{-1}(F) \subseteq V$. Since$f$ s-co$\acute{c}$-open then $f(U), f(V)$ disjoint s-coc-open sets in Y and since f onto then $f\left(f^{-1}(F)\right) \subseteq F$ .Then $f(x) \subseteq f(U) and F \subseteq f\ (V)$
Then Y is s-coc-regular.

Proposition (3.22)

A space X is s-coc^{*}-regular if and only if for all $x \in X$ and all U s-coc-open set such that $x \in U$ there exists W s-coc-open set such that $x \in W \subseteq \overline{W}^{s-coc} \subseteq U$

Proof

Let X is s-coc^{*}-regular,$x \in X$ and U s-coc-open set such that $x \in U$ .Then U^{c}s-coc-closed set and $x \notin U^{c}$. Since X is s-coc^{*}-regular then there exist disjoint s-coc-open sets V, W such that x $\in W, U^{c} \subseteq V$. Hence $x \in W \subseteq \overline{W}^{s-coc} \subseteq \overline{V^{c}}^{s-coc} \subseteq V^{c} = U$
Conversely

Let for all $x \in X$,U s-coc-open set such that $x \notin U$ .Such that W s-coc-open set such that $x \in W \subseteq \overline{W}^{s-coc} \subseteq U$ .Let $x \in X$, F s-coc-closed set in X such that $x \notin F$. Then F^{c}s-coc-open set and $x \in F^{c}$.Then $x \in W \subseteq \overline{W}^{s-coc} \subseteq F^{c}$ .Then $x \in W, F \subseteq \left(\overline{W}^{s-coc}\right)^{c}$ .Since $\overline{W}^{s-coc}$ s-coc-closed set, then $\left(\overline{W}^{s-coc}\right)^{c}$ s-coc-open set and$W, \left(\overline{W}^{s-coc}\right)^{c}$disjoint . Then X s-coc^{*}-regular .

Proposition (3.23)

If $f: X \longrightarrow Y$ s-co$\acute{c}$-homeomorphism. Then Y s-coc^{*}-regular if and only if X is s-coc^{*}-regular

Proof

Let X is s-coc^{*}-regular and $y \in Y\ and\ F$ s-coc-closed set in Y such that $y \notin F$ .Since f onto then there exists $x \in X$ such that $f(x) = y$ .Then $x = f^{-1}(y)$ .Since f s-co$\acute{c}$-continuous then $f^{-1}(F)$ closed set in X and $x \notin f^{-1}(F)$ .Since X is s-coc^{*}-regular then there existU, V disjoint s-coc-open sets such that $x \in U, f^{-1}(F) \subseteq V$ .Since f s-coc′-open then $f(U), f\ (V)$ disjoint s-coc-open sets in Y and $f(x) \notin f(U)$, $f\left(f^{-1}(F)\right) \subseteq V$. Since f be onto then $f\left(f^{-1}(F)\right) = F$ .Then $y \notin f(U), F \subseteq V$Then Y s-coc^{*}-regular
Conversely

Let Y s-coc^{*}-regular and $x \in X$, F s-coc-closed set in X such that $x \notin F$ then $y = f(x) \notin f(F)$ .Since f s-co$\acute{c}$-closed then $f(F)$ s-coc-open in Y and $y \notin f(F)$. Since Y s-coc^{*}-regular then there existU, V disjoint s-coc-open sets such that $y \in U, f\ (F) \subseteq V$
Since f s-coc′-continuous then$f^{-1}(U), f^{-1}\ (V)$ disjoint s-coc-open sets in$X$ such that$=$ $f^{-1}(y) \in f^{-1}(U), f^{-1}\left(f(F)\right) \subseteq f^{-1}\ (V)$. Since f one to one then $F \subseteq f^{-1}\left(f(F)\right)$ .Then X s-coc^{*}-regular

Definition (3.18)

A space X is said to be s-coc*-normal if for all $x \in X$ and all A, B disjoint s-coc-closed sets in X there exist U, V disjoint s-coc-open sets such that $A \subseteq U, B \subseteq V$

Proposition (3.24)

Every s-coc^*-normal is s-coc-normal

Proof

Let A, B disjoint closed sets in X .Then A, B are disjoint s-coc-closed sets in X .Since X s-coc^*-regular then there exist U, V disjoint s-coc-open sets such that $A \subseteq U, B \subseteq V$.Then X s-co$c$-normal

Proposition (3.25)

A space X is s-coc-normal if and only if for all s-coc-closed set $D \subseteq X$ and all U s-coc-open set in X such that $D \subseteq U$ there exists V s-coc-open sets such that D $\subseteq V \subseteq \overline{V}^{s-coc} \subseteq U$.

Proof

Let X s-coc^*-normal and $D \subseteq X$ such that D s-coc-closed and U s-coc-open in X such that $D \subseteq U$ .Then D, U^cdisjoint s-coc-closed sets. Since X s-coc^*-normal then there exist W, V disjoint s-coc-open sets such that D $\subseteq V, U^c \subseteq W$. Then $D \subseteq V \subseteq \overline{V}^{s-coc} \subseteq \overline{W^c}^{s-coc} \subseteq U$

Conversely

Let D_1, D_2 disjoint s-coc-closed sets in X.Then ${D_2}^c$ open set and $D_1 \subseteq {D_2}^c$. Then there exists s-coc-open set V .Such that $D \subseteq V \subseteq \overline{V}^{s-coc} \subseteq {D_2}^c$ .Then $D \subseteq V$, ${D_2}^c \subseteq \left(\overline{V}^{s-coc}\right)^c$.Since $V, \left(\overline{V}^{s-coc}\right)^c$ are disjoint s-coc-open sets .Then X s-coc^*-normal.

Proposition (3.26)

If X s-coc^*-normal and s-coc-T_1. Then X s-coc^*-regular .

Proof

Let $x \in X$ and W s-coc-open set such that $x \in W$. Since X $is\ s{-}coc{-}T_1$ then $\{x\}$ s-coc-closed set by proposition (3.1) .Then $\{x\} \subseteq W$. Since X s-coc* -normal then there exists V s-coc-open set, such that $\{x\} \subseteq V \subseteq \overline{V}^{s-coc} \subseteq W$ .Then X s-coc^*-regular by proposition (3.22)

Proposition (3.27)

If $f: X \longrightarrow Y$ onto super s-coc-open, continuous and X s-coc^*-normal, then Y is normal

Proof

Let A, B disjoint closed sets in Y .Since f continuous then $f^{-1}(A), f^{-1}(B)$ disjoint closed sets in X . Since X s-coc^*-normal then there exists two disjoint s-coc-open sets U, V such that $f^{-1}(A) \subseteq U, f^{-1}(B) \subseteq V$. Since f super s-coc-open then $f(U), f(V)$ disjoint open sets in Y and $f\left(f^{-1}(A)\right) \subseteq f(U), f\left(f^{-1}(B)\right) \subseteq f(V)$. Since f onto then A$\subseteq f(U), B \subseteq f(V)$. Then Y is normal

Proposition (3.28)

Let X s-coc^*-normal and $f: X \longrightarrow Y$ bijective s-coc-continuous and s-co$\acute{c}$-open, then Y is s-coc-normal

Proof

Let A, B disjoint closed sets in Y .Since f continuous then $f^{-1}(A), f^{-1}(B)$ disjoint closed sets in X . Since X s-coc^*-normal then there exists two disjoint s-coc-open sets U, V such that $f^{-1}(A) \subseteq U, f^{-1}(B) \subseteq V$. Since f s-co$\acute{c}$-open then $f(U), f(V)$ disjoint s-coc-open sets in Y and$\left(f^{-1}(A)\right) \subseteq f(U), f\left(f^{-1}(B)\right) \subseteq f(V)$. Since f bijective then A$\subseteq f(U), B \subseteq f\ (V)$. Then Y is s-coc-normal

Proposition (3.29)
If $f: X \longrightarrow Y$ s-co$\acute{c}$-homeomorphism. Then Y is s-coc^{*}-normal if and only if X s-coc^{*}-normal

Proof
Let X s-coc^{*}-normal and A, B disjoint s-coc-closed sets in Y .Since f s-co$\acute{c}$-continuous then $f^{-1}(A), f^{-1}(B)$ disjoint s-coc-closed sets in X . Since X s-coc^{*}-normal then there exists two disjoint s-coc-open sets U, V such that $f^{-1}(A) \subseteq U, f^{-1}(B) \subseteq V$. Since f s-co$\acute{c}$-open then $f(U), f(V)$ disjoint open sets in Y and$f\left(f^{-1}(A)\right) \subseteq f(U), f\left(f^{-1}(B)\right) \subseteq f(V)$ .Since f bijective then A $\subseteq f(U), B \subseteq f\ (V)$. Then Y is s-coc* -normal.
Conversely
Let Y is s-coc^{*}-normal and A, B disjoint s-coc-closed sets in X . Since f s-co$\acute{c}$-open then $f(A), f(B)$ disjoint s-coc-closed sets in Y .Since Y is s-coc^{*}-normal then there exists two disjoint s-coc-open sets U, V in Y such that $f(A) \subseteq U, f(B) \subseteq V$. Since f s-co$\acute{c}$-continuous then $f^{-1}(U), f^{-1}(V)$ are disjoint s-coc-open sets in X such that $f^{-1}\left(f(A)\right) \subseteq f^{-1}(U), f^{-1}\left(f(B)\right) \subseteq f^{-1}(V)$ .Since f onto then $A \subseteq f^{-1}(U), B \subseteq f^{-1}(V)$ .Then X s-coc^{*}-normal space

Definition (3.19)
A space X is said to be locally s-coc-regular if for all $x \in X$ and all F locally closed set such that $x \notin F$ there exists A, B disjoint s-coc-open sets such that $x \in A, F \subseteq B$

Definition (3.20)
A space X is said to be locally s-coc^{*}-regular if for all $x \in X$ and all F locally s-coc-closed set such that $x \notin F$ there exists disjoint s-coc-open sets U, V such that $\in U, F \subseteq V$.

Proposition (3.30)
Every locallys-coc^{*}-regular is s-coc^{*}-regular
Proof
Let $x \in X$, X locallys-coc^{*}-regular, let A is s-coc-closed set in X such that $x \notin A$. Since A= $X \cap A$and X $\in \tau$ thus A is locally s-coc-closed set. Since Xlocally s-coc^{*}-regular then there exists U, V disjoint s-coc-open sets such that $x \in U, A \subseteq V$. Then X is s-coc^{*}--regular

Remark (3.7)
Every locally s-coc^{*}-regular is s-coc regular

Definition (3.21)
A space X is said to be locally s-coc-normal if for every all A, B disjoint locally closed sets there exists U, V disjoint s-coc-open sets such that $A \subseteq U, B \subseteq V$

Definition (3.22)
A space X is said to be locally s-coc^{*}-normalif for every all A, B disjoint locally s-coc-closed sets there exists U, V disjoint s-coc-open sets such that $A \subseteq U, B \subseteq V$

Proposition (3.31)
Every locally s-coc^{*}–normal is s-coc^{*}-normal
Proof
Let $A, B$are disjoint s-coc-closed sets in X. Since $A = \mathrm{X} \cap \mathrm{A}$, $B = \mathrm{X} \cap \mathrm{B}$ and X $\in \tau$, thus A , B are disjoint locally s-coc-closed sets .Since X is locally s-coc^{*}–normal . Then there exists disjoint s-coc-open sets U, V such thatA $\subseteq U$, B $\subseteq V$ .Therefore X is s-coc^{*}-normal .

Remark (3.8)

Every locally s-coc^*-normal is s-coc^*-normal .

Proposition (3.32)

Every locally s-coc^*-regular is locally s-coc-regular

Proof

Let X is locally s-coc^*-regular ,let F locally closed set in X and $x \notin F$ then by proposition (1.16) we get F is locally s-coc-closed set .Since X is s-coc^*-regular then there exists U, V disjoint s-coc-open sets such that $x \in U, F \subseteq V$. Then X is locally s-coc-regular

Proposition (3.33)

If X locally s-coc^*-regular and $f: X \longrightarrow Y$ continuous and s-co$\acute{c}$-homeomorphism , then Y is locally s-coc^*-regular

Proof

Let $y \in Y$, A locally s-coc-closed set in Y such that $y \notin A$ .Since f onto then there exists $x \in X$ such that $f(x) = y$. Then x= $f^{-1}(y)$. Since f continuous and s-co$\acute{c}$-contionuous then $f^{-1}(A)$ locally s-coc-closed set in X by proposition (2.11) and $x \notin f^{-1}(A)$ in X . Since X locally s-coc*-regular then there exists U, V disjoint s-coc-open sets such that x $\in U, f^{-1}(A) \subseteq V$. Since f s -co$\acute{c}$-open then $f(U), f(V)$ disjoint s-coc-open sets in $Y$and y $= f(x) \in f(U), f\left(f^{-1}(A)\right) \subseteq f(V)$. Since f onto then y $\in f(U), A \subseteq f(V)$. Then Y is locally s-coc^*-regular

Proposition (3.34)

Every locally s-coc^*-normal is locally s-coc-normal

Proof by proposition (2.16) .

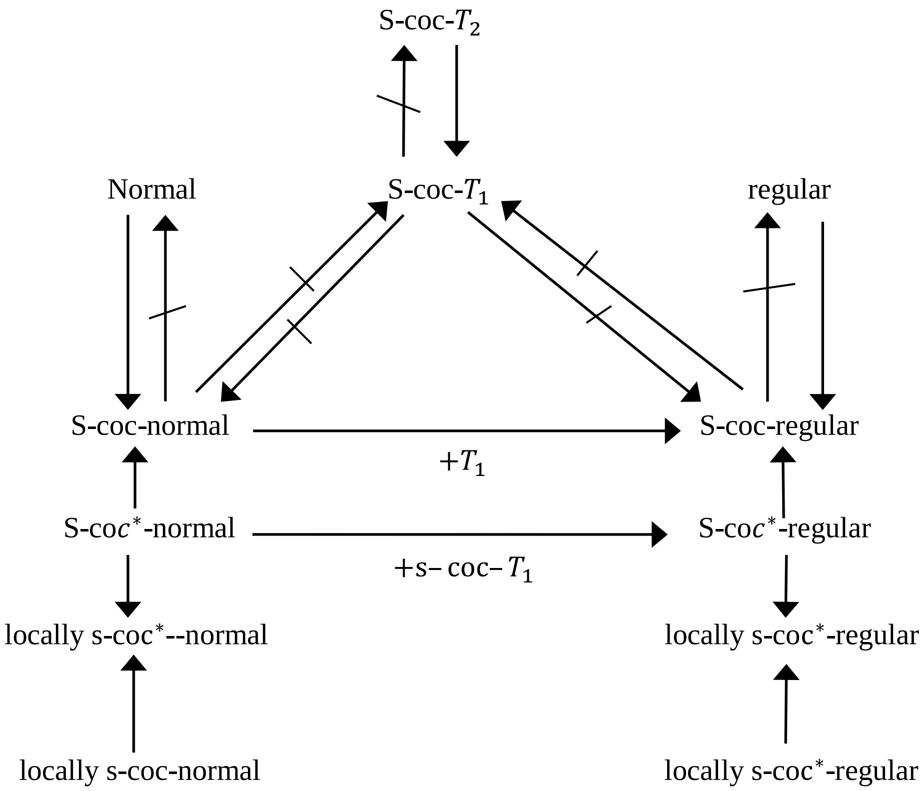
Proposition (3.35)

If X locally s-coc^*-normal and $f: X \longrightarrow Y$ continuous and s-co$\acute{c}$-homeomorphism . Then Y is locally s-coc^*-normal

Proof

Let A, Bdisjoint locally s-coc-closed sets in . Since f continuous, s-co$\acute{c}$-continuous then $f^{-1}(A), f^{-1}(B)$ locally s-coc-closed set in X by proposition (2.11) .Since X locally s-coc^*-normal then there exists disjoint s-coc-open sets U, V such that $f^{-1}(A) \subseteq U, f^{-1}(B) \subseteq V$. Since f s-co$\acute{c}$-open then $f(U), f(V)$ disjoint s-coc-open sets in $Y$and $\left(f^{-1}(A)\right) \subseteq f(U), f\left(f^{-1}(B)\right) \subseteq f(V)$. Since f onto then $A \subseteq f(U), B \subseteq f(V)$

Then Y is locally s-coc^*-normal


S-coc-T_2
Normal
S-coc-T_1
regular
S-coc-normal
S-coc-regular
$+T_1$
S-coc^*-normal
S-coc^*-regular
$+$s$-$ coc$-T_1$
locally s-coc*--normal
locally s-coc*-regular
locally s-coc-normal
locally s-coc*-regular

Reference

[1] N. Bourbaki , Elements of Mathematics “General topology ” Chapter 1-4 , Spring Vorlog , Belin , Heidelberg , New- York , London , Paris , Tokyo 2 nd Edition (1989).

[2] S.G.Crossley and S.K.Hildebr and “On Semi Closure” TaxasJ.Sci. , 22 , (1971).

[3] M. C. Gemignani , “Elementary Topology” Reading Mass.Addison-wesley Publishing Co.Inc. , 1967 .

[4] E. Hatir and T.Noiri , Decomposition of Continuity and Complete Continuity , Acta Math Hunger , 133(4)(2006) 281-287 .

[5] F. H. Jasim “On Compactness Via Cocompact open sets” M.SC. Thesis University of Al-Qadisiya ,college of Mathematics and computer science , 2014.

[6] N.Levine “Semi-open sets and Semi Continuity in Topological space “Amer Math. Monthy ,70, (1963).

[7]A.G.Naoum and R.A.H.Al-Abdulla "On paracompact space using semi-open sets "Al-Qadisiya jmournalfor computer Since and Mathematics , Vol 2 ,No .1 (2010) .

[8] N.J.Pervin , “Foundation of General Topology ” Academic Press , New York , 1964.

[9] S.Alghour and S.Samarah “ Cocompact open sets and Continuity ” Abstract and applied analysis , Volume 2012 , Article ID 548612 , 9 Pages , 2012 .

[10] J.N.Sharma “ Topology ” , published by Krishna Prakasha Mandir , Meerut (U.P) Printed at Manoj printers , Meerut (1977) .

[11] S. Willarad ,General Topology , Addison Wesley Publishing Company Reading Mass 1970

حول بديهيات الفصل من النمط

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المستخلص

في هذا البحث قدمنا نوع جديد من بديهيات الفصل اسميناها s-coc-separation axioms . لقد ظهرت خلال البحث مفاهيم جديدة منها التي تم شرحها ضمن المجموعات المفتوحة من النمط s-coc. لقد تناولنا انواع من الدوال المستمرة ووضحنا العلاقة بينها .وتناولنا تعاريف بديهيات الفصل من النمط s-coc ووضحنا خواصها و العلاقات بينها .