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On Semiprime Gamma Near-Rings with Perpendicular Generalized 3-Derivations

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Abstract:

In this paper , we introduce the notion of perpendicular generalized 3-derivations in semiprime gamma near-rings and present several necessary and sufficient conditions for generalized 3-derivations on semiprime gamma near-rings to be perpendicular .

KeyWords: Semiprime Γ-near-ring , 3-derivations , Generalized 3-derivations , perpendicular 3-derivations , perpendicular generalized 3-derivations .

Mathematics Subject Classification: 16A70, 16N60 , 16W25 .

Introduction

This paper consists of two sections . In section one , we recall some known definitions and necessary lemmas that we will use it later in this paper . In section two , we begin by introducing definition of perpendicular generalized 3-derivations in Γ-near-rings. Furthermore , several conditions are given to make the two generalized 3-derivations perpendicular .

1. Basic Concept

Definition 1.1:[2]

A right near-ring (resp. a left near-ring) is a nonempty set N equipped with two binary operations + and . such that

(i)(N , +) is a group (not necessarily abelian)

(ii) (N , .) is a semigroup

(iii) For x , y , z ∈N , we have

(x + y)z = xz + yz (resp. z(x + y) = zx + zy)

Example 1.2 :[2]

Let G be a group (not necessarily abelian) then all mapping of G into itself form a right near-ring M(G) with regard to point wise addition and multiplication by composite .

Definition 1.3:[2]

Let M and Γ be additive abelian groups . If there exists a mapping M x Γ x M → M : (a , α , b) → a α b which satisfies the conditions : for every a , b , c ∈ M , α , β ∈ Γ

(i)(a + b) α c = a αc + b α c

a (α + β)b = a αb + a βb

a α (b + c) = a αb + a αc

(ii)(a αb) β c = a α (b βc)

Then M is called a Γ-ring .

Example 1.4 :[2]

Let R be a ring , the additive abelian groups M = M_{2x3}(R) and Γ = M_{3x2}(R) denotes the sets of all 2x3 matrices over R and 3x2 matrices over R respectively . Then M is Γ-ring .

Definition 1.5:[5]

A Γ-near-ring is a triple (N , + , Γ) where

(i)(N , +) is a group (not necessarily abelian)

(ii) Γ is a non-empty set of binary operations on N such that for each α ∈ Γ , (N , + , α) is a left near-ring .

(iii)xα(y βz) = (x αy) βz for all x ,y, z ∈ N , α , β ∈ Γ .

Definition 1.6:[5]

Let W be a Γ-near-ring , the set W_0 = { x∈ W: 0αx = 0 , α ∈ Γ } is said to be zero-symmetric part of W . A Γ-near-ring W is called zero-symmetric if W = W_0 .

Definition 1.7:[5]

A Γ-near-ring W is said to be a prime Γ-near-ring when W satisfy the following for a, b ∈ W , a Γ W Γ b = {0} implies a = 0 or b = 0

Definition 1.8:[5]

A Γ-near-ring W is said to be a semiprime when W satisfy the following for a ∈ W , a Γ W Γ a = {0} implies a = 0 .

Definition 1.9:[5]

A Γ-near-ring W is said to be 2-torsion free if for all x∈ W , 2x = 0 implies x = 0 .

Definition 1.10:[3]

Let W be a Γ-near-ring . An additive map T : W → W is said to be a derivation if T(a αb) =T(a) αb + a αT(b) for every a ,b ∈ W , α ∈ Γ.

Definition 1.11:[3]

Let W be a Γ-near-ring and D : W → W be an additive map . If there exists a derivation d : W → W such that D (x αy) = D(x) αy + x α d(y) holds for all x ,y ∈ W , α ∈ Γ , then D is called a generalized derivation.

Definition 1.12:[1]

Suppose that W is a near-ring . An 3-additive mapping d :W x W x W → W is called 3-derivation if the relations :

$d(s_1s_1^{/}, s_2 , s_3) =d(s_1, s_2 , s_3)s_1^{/} + s_1\, d(s_1^{/}, s_2 , s_3)$

$d(s_1, s_2s_2^{/} , s_3) =d(s_1, s_2 , s_3)s_2^{/} + s_2\, d(s_1, s_2^{/} , s_3)$

$d(s_1, s_2, s_3s_3^{/}) = d(s_1, s_2 , s_3)s_3^{/} + s_3\, d(s_1, s_2 , s_3^{/})$

hold for all $s_1 , s_1^{/} , s_2 , s_2^{/} , s_3 , s_3^{/}$ ∈ W .

Example 1.13 :[1]

Let S be a commutative near-ring .

Let us define

$W=\left\{\begin{pmatrix} r & u \\ 0 & 0\end{pmatrix}: r, u, 0 \in S\right\}$.

And d : WxWx W→ W

$d\left(\begin{pmatrix} r_1 & u_1 \\ 0 & 0\end{pmatrix}, \begin{pmatrix} r_2 & u_2 \\ 0 & 0\end{pmatrix}, \begin{pmatrix} r_3 & u_3 \\ 0 & 0\end{pmatrix}\right) = \begin{pmatrix} 0 & r_1 r_2 r_3 \\ 0 & 0\end{pmatrix}$

Then d is 3-derivation of W .

Definition 1.14:[1]

Suppose that W is a near-ring and d be 3-derivation of W . An 3-additive mapping f : WxWxW→ W is said to be generalized 3-derivation of W associated with d if the relations

$f(s_1s_1', s_2, s_3) = f(s_1, s_2, s_3)s_1' + s_1 d(s_1', s_2, s_3)$

$f(s_1, s_2s_2', s_3) = f(s_1, s_2, s_3)s_2' + s_2 d(s_1, s_2', s_3)$

$f(s_1, s_2, s_3s_3') = f(s_1, s_2, s_3)s_3' + s_3 d(s_1, s_2, s_3')$

hold for all $s_1, s_1', s_2, s_2', s_3, s_3' \in W$.

Example 1.15 :[1]

Let S be a commutative near-ring .

Let us define

$W=\left\{\begin{pmatrix} 0 & r \\ 0 & u\end{pmatrix}: r, u, 0 \in S\right\}$.

And d, f : WxWxW→ W ,

$d\left(\begin{pmatrix} 0 & r_1 \\ 0 & u_1\end{pmatrix}, \begin{pmatrix} 0 & r_2 \\ 0 & u_2\end{pmatrix}, \begin{pmatrix} 0 & r_3 \\ 0 & u_3\end{pmatrix}\right) = \begin{pmatrix} 0 & r_1 r_2 r_3 \\ 0 & 0\end{pmatrix}$

$f\left(\begin{pmatrix} 0 & r_1 \\ 0 & u_1\end{pmatrix}, \begin{pmatrix} 0 & r_2 \\ 0 & u_2\end{pmatrix}, \begin{pmatrix} 0 & r_3 \\ 0 & u_3\end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & u_1 u_2 u_3\end{pmatrix}$

Then f is a generalized 3-derivation of W .

Lemma 1.16 :[4]

Let W be a 2-torsion free semiprime Γ-near-ring and a , b ∈ W. When

(i)$a\alpha x\beta b = 0$ for all x∈ W and $\alpha, \beta \in \Gamma$.

(ii) $b\alpha x\beta a = 0$ for all x∈ W and $\alpha, \beta \in \Gamma$.

(iii) $a\alpha x\beta b + b\alpha x\beta a = 0$ for all x∈ W and $\alpha, \beta \in \Gamma$. are equivalent

When one of the above is satisfied , implies a Γ b = b Γ a = 0 .

Lemma 1.17 :[4]

Assume that W is a Γ-near-ring and D be a generalized derivation of W . When the next conditions are satisfied :

(i)$(D(x)\alpha y + x\alpha d(y))\beta z = D(x)\alpha y\beta z + x\alpha d(y)\beta z$

For every x ,y, z ∈ W and $\alpha, \beta \in \Gamma$.

(ii)$(d(x)\alpha y + x\alpha d(y))\beta z = d(x)\alpha y\beta z + x\alpha d(y)\beta z$

for every x ,y, z ∈ W , $\alpha, \beta \in \Gamma$.

2. Perpendicular Generalized 3-Derivations

First we introduce the basic definition in this paper

Definition 2.1:

Let W be a Γ-near-ring and P , Q be two generalized 3- derivations of W . P and Q are called perpendicular if next relation $P(s_1, s_2, s_3)\,\Gamma\, W\,\Gamma\, Q(n_1, n_2, n_3) = 0 = Q(n_1, n_2, n_3)\Gamma\, W\,\Gamma\, P(s_1, s_2, s_3)$ holds for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.

Lemma 2.2:

Assume that W is a 2-torsion free semiprime Γ-near-ring and P , Q are two generalized 3-derivations of W with associated 3-derivations p and q of W respectively . When p and q are perpendicular , so next conditions are satisfied :

(i) $P(s_1, s_2, s_3)\Gamma Q(n_1, n_2, n_3) = Q(s_1, s_2, s_3)\Gamma P(n_1, n_2, n_3) = 0$

For every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.

(ii)p and q are perpendicular and $p(s_1, s_2, s_3)\,\Gamma Q(n_1, n_2, n_3) = Q(n_1, n_2, n_3)\,\Gamma\, p(s_1, s_2, s_3) = 0$

for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.

(iii)q and P are perpendicular and $q(s_1, s_2, s_3)\Gamma P(n_1, n_2, n_3) = P(n_1, n_2, n_3)\Gamma q(s_1, s_2, s_3) = 0$

for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.

(iv)p and q are perpendicular and $p(q(s_1, s_2, s_3), n_2, n_3) = 0$ for every $s_1, s_2, s_3, n_2, n_3 \in W$

(v) $p(Q(s_1, s_2, s_3), n_2, n_3) = Q(p(s_1, s_2, s_3), n_2, n_3) = 0$

for all $s_1, s_2, s_3, n_2, n_3 \in W$, and $q(P(s_1, s_2, s_3), n_2, n_3) = P(q(s_1, s_2, s_3), n_2, n_3) = 0$

for all $s_1, s_2, s_3, n_2, n_3 \in W$.

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(vi) $Q(P(s_1, s_2, s_3), n_2, n_3) = P(Q(s_1, s_2, s_3), n_2, n_3) = 0$

for all $s_1, s_2, s_3, n_2, n_3 \in W$.

Proof:

(i)Since P and Q are perpendicular , implies that

$P(s_1, s_2, s_3)\, \alpha\, s\, \beta\, Q(n_1, n_2, n_3) = 0$ for every $s, s_1, s_2, s_3, n_1, n_2, n_3 \in W$ and $\alpha, \beta \in \Gamma$.

By Lemma 1.17 we get

$P(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3) = Q(s_1, s_2, s_3)\, \alpha\, P(n_1, n_2, n_3) = 0$ for all $s_1, s_2, s_3, n_1, n_2, n_3 \in W$ and $\alpha \in \Gamma$.

(ii) by (i) we get

$P(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3) = 0$

for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha \in \Gamma$.

Replacing s_1 by $s_1^{/}\, \beta\, s_1$, where $s_1^{/} \in W$ and $\beta \in \Gamma$ in previous equation and using Lemma 1.17 we get

$0 = P(s_1^{/}\, \beta\, s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

$= [P(s_1^{/}, s_2, s_3)\, \beta\, s_1 + s_1^{/}\, \beta\, p(s_1, s_2, s_3)]\, \alpha\, Q(n_1, n_2, n_3)$

$= P(s_1^{/}, s_2, s_3)\, \beta\, s_1\, \alpha\, Q(n_1, n_2, n_3)$

$+ s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

$= s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

for all $s_1^{/}, s_1, s_2, s_3, n_1, n_2, n_3 \in W$ and $\alpha, \beta \in \Gamma$.

Since W is semiprime we get

$p(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3) = 0$

for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$ and $\alpha \in \Gamma$.

Now replacing s_1 by $s_1\, \beta\, s_1^{/}$, where $s_1^{/} \in W$ and $\beta \in \Gamma$ in previous equation and using Lemma 1.17 (ii) , we get

$0 = p(s_1\, \beta\, s_1^{/}, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

$= [p(s_1, s_2, s_3)\, \beta\, s_1^{/} + s_1\, \beta p(s_1^{/}, s_2, s_3)]\, \alpha\, Q(n_1, n_2, n_3)$

$= p(s_1, s_2, s_3)\, \beta\, s_1^{/}\, \alpha\, Q(n_1, n_2, n_3)$

$+ s_1\, \beta p(s_1^{/}, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

$= p(s_1, s_2, s_3)\, \beta\, s_1^{/}\, \alpha\, Q(n_1, n_2, n_3)$

for all $s_1^{/}, s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha, \beta \in \Gamma$.

By Lemma 1.16 we obtain

$p(s_1, s_2, s_3)\, \Gamma\, Q(n_1, n_2, n_3) = Q(n_1, n_2, n_3)\, \Gamma\, p(s_1, s_2, s_3) = 0$

for all $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.

(iii) The proof is similar to (ii)

(iv) By (i) we get

$P(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3) = 0$

for all $s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha \in \Gamma$.

Replacing s_1 by $s_1^{/}\, \beta\, s_1$, where $s_1^{/} \in W$ and $\beta \in \Gamma$ in previous equation and using Lemma 1.17 we get

$0 = P(s_1^{/}\, \beta\, s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

$= [P(s_1^{/}, s_2, s_3)\, \beta\, s_1 + s_1^{/}\, \beta\, p(s_1, s_2, s_3)]\, \alpha\, Q(n_1, n_2, n_3)$

$= P(s_1^{/}, s_2, s_3)\, \beta\, s_1\, \alpha\, Q(n_1, n_2, n_3)$

$+ s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

$= s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, Q(n_1, n_2, n_3)$

For every $s_1^{/}, s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha, \beta \in \Gamma$.

Replacing n_1 by $n_1 \delta\, n_1^{/}$, where $n_1^{/} \in W$ and $\delta \in \Gamma$ in previous equation and using (ii) , we get

$0 = s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, Q(n_1 \delta\, n_1^{/}, n_2, n_3)$

$= s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha [Q(n_1, n_2, n_3)\, \delta\, n_1^{/} + n_1 \delta\, q(n_1^{/}, n_2, n_3)$

$= s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha Q(n_1, n_2, n_3)\, \delta\, n_1^{/}$

$+ s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, n_1 \delta\, q(n_1^{/}, n_2, n_3)$

$= s_1^{/}\, \beta\, p(s_1, s_2, s_3)\, \alpha\, n_1 \delta\, q(n_1^{/}, n_2, n_3)$

for all $s_1^{/}, s_1, s_2, s_3, n_1, n_1^{/}, n_2, n_3 \in W$, $\alpha, \beta, \delta \in \Gamma$.

Semiprimeness of W implies

$p(s_1, s_2, s_3)\, \alpha\, n_1 \delta\, q(n_1^{/}, n_2, n_3) = 0$

for every $s_1, s_2, s_3, n_1, n_1^{/}, n_2, n_3 \in W$, $\alpha, \delta \in \Gamma$.

That is p and q are perpendicular, then we have

$0 = p(p(v_1, v_2, v_3)\, \alpha\, n_1\, \beta\, q(s_1, s_2, s_3), n_2, n_3)$

$= p(p(v_1, v_2, v_3), n_2, n_3)\, \alpha\, n_1\, \beta\, q(s_1, s_2, s_3)$

$+ p(v_1, v_2, v_3)\, \alpha\, p(n_1\, \beta\, q(s_1, s_2, s_3), n_2, n_3)$

for all $s_1, s_2, s_3, n_1, n_2, n_3, v_1, v_2, v_3 \in W$, $\alpha, \beta \in \Gamma$.

Since p , q are perpendicular , we have

$0 = p(v_1, v_2, v_3)\, \alpha\, p(n_1\, \beta\, q(s_1, s_2, s_3), n_2, n_3)$

$= p(v_1, v_2, v_3)\, \alpha\, p(n_1, n_2, n_3)\, \beta\, q(s_1, s_2, s_3)$

$+ p(v_1, v_2, v_3)\, \alpha\, n_1\, \beta\, p(q(s_1, s_2, s_3), n_2, n_3)$

Since p , q are perpendicular , we conclude

$p(v_1, v_2, v_3)\, \alpha\, n_1\, \beta\, p(q(s_1, s_2, s_3), n_2, n_3) = 0$

for all $s_1, s_2, s_3, n_1, n_2, n_3, v_1, v_2, v_3 \in W$, $\alpha, \beta \in \Gamma$.

Replacing v_1 by $q(s_1, s_2, s_3)$ and v_2 by n_2, v_3 by n_3, implies
$p(q(s_1, s_2, s_3), n_2, n_3)\,\Gamma\, W\, \Gamma\, p(q(s_1, s_2, s_3), n_2, n_3) = \{0\}$
for all $s_1, s_2, s_3, n_2, n_3 \in W$. Semipimeness of W implies that
$p(q(s_1, s_2, s_3), n_2, n_3) = 0$ for all $s_1, s_2, s_3, n_2, n_3 \in W$.
Now we prove part 5 and part 6
Using part 2 and part 4, we have
$0 = Q(p(s_1, s_2, s_3)\,\alpha\, s\,\beta\, Q(n_1, n_2, n_3), n_2, n_3)$
$= Q(p(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s\,\beta\, Q(n_1, n_2, n_3)$
$+ p(s_1, s_2, s_3)\,\alpha q\,(s\beta\, Q(n_1, n_2, n_3), n_2, n_3)$
$= Q(p(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s\,\beta\, Q(n_1, n_2, n_3)$
for all $s, s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha, \beta \in \Gamma$.
Replacing n_1 by $p(s_1, s_2, s_3)$, we get
$Q(p(s_1, s_2, s_3), n_2, n_3)\,\alpha\, W\,\beta\, Q(p(s_1, s_2, s_3), n_2, n_3) = \{0\}$
Semiprimeness of W implies that
$Q(p(s_1, s_2, s_3), n_2, n_3) = 0$ for all $s_1, s_2, s_3, n_2, n_3 \in W$.
Similarly, we see that since
$p(Q(s_1, s_2, s_3)\,\alpha\, s\,\beta\, p(n_1, n_2, n_3), n_2, n_3) = 0$
$P(q(s_1, s_2, s_3)\,\alpha\, s\,\beta\, P(n_1, n_2, n_3), n_2, n_3) = 0$
$q(P(s_1, s_2, s_3)\,\alpha\, s\,\beta\, q(n_1, n_2, n_3), n_2, n_3) = 0$
$P(Q(s_1, s_2, s_3)\,\alpha\, s\,\beta\, P(n_1, n_2, n_3), n_2, n_3) = 0$
$Q(P(s_1, s_2, s_3)\,\alpha\, s\,\beta\, Q(n_1, n_2, n_3), n_2, n_3) = 0$
Hold for every $s, s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha, \beta \in \Gamma$,
we get
$p(Q(s_1, s_2, s_3), n_2, n_3) = 0$
$P(q(s_1, s_2, s_3), n_2, n_3) = 0$
$q(P(s_1, s_2, s_3), n_2, n_3) = 0$
$P(Q(s_1, s_2, s_3), n_2, n_3) = 0$ and
$Q(P(s_1, s_2, s_3), n_2, n_3) = 0$
for all $s_1, s_2, s_3, n_2, n_3 \in W$.

Theorem 2.3 :

Assume that W is a 2-torsion free semiprime Γ-near-ring and P, Q are two generalized 3-derivations of W with associated 3-derivations p and q of W respectively, then the next conditions are satisfied :
(i) P and Q are perpendicular.
(ii) $P(s_1, s_2, s_3)\Gamma Q(n_1, n_2, n_3) = p(s_1, s_2, s_3)\Gamma Q(n_1, n_2, n_3) = 0$
for all $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.
(iii) $P(s_1, s_2, s_3)\Gamma Q(n_1, n_2, n_3) = p(s_1, s_2, s_3)\Gamma q(n_1, n_2, n_3) = 0$
And $p(Q(s_1, s_2, s_3), n_2, n_3) = p(q(s_1, s_2, s_3), n_2, n_3) = 0$
for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.
(iv) $P(Q(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3) = P(Q(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s_1^{/} + s_1\,\alpha\, p(q(s_1^{/}, s_2, s_3), n_2, n_3)$
and
$P(s_1, s_2, s_3)\Gamma\, Q(n_1, n_2, n_3) = 0$
for all $s_1, s_1^{/}, s_2, s_3, n_1, n_2, n_3 \in W$ and $\alpha \in \Gamma$.

Proof :

(i)⇒(ii) and (i) ⇒(iii) are proved in Lemma 2.2 (i), (ii), (iv) and (v).
On the other hand, (i) ⇒(iv) is obtained from Lemma 2.2 (i), (iv) and (vi). (ii) ⇒(i) we have
$P(s_1, s_2, s_3)\Gamma\, Q(n_1, n_2, n_3) = p(s_1, s_2, s_3)\,\Gamma\, Q(n_1, n_2, n_3) = 0$
for all $s_1, s_2, s_3, n_1, n_2, n_3 \in W$.
Now replacing s_1 by $s_1\,\beta\, s_1^{/}$, where $s_1^{/} \in W$ and $\beta \in \Gamma$ in previous equation, we get
$P(s_1\,\beta\, s_1^{/}, s_2, s_3)\Gamma\, Q(n_1, n_2, n_3) = 0$
for all $s_1^{/}, s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\beta \in \Gamma$.
Using hypothesis and Lemma 1.17 in last equation, we get
$0 = [P(s_1, s_2, s_3)\,\beta\, s_1^{/} + s_1\,\beta p(s_1^{/}, s_2, s_3)]\,\alpha\, G(n_1, n_2, n_3)$
$= P(s_1, s_2, s_3)\,\beta\, s_1^{/}\,\alpha\, Q(n_1, n_2, n_3)$
$+ s_1\,\beta\, p(s_1^{/}, s_2, s_3)\,\alpha\, Q(n_1, n_2, n_3)$
$= P(s_1, s_2, s_3)\,\beta\, s_1^{/}\,\alpha\, Q(n_1, n_2, n_3)$
for all $s_1^{/}, s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha, \beta \in \Gamma$.
Hence Lemma 1.16 obtain the result.
(iii) ⇒(i) we have
$p(Q(s_1, s_2, s_3), n_2, n_3) = p(q(s_1, s_2, s_3), n_2, n_3) = 0$
for all $s_1, s_2, s_3, n_2, n_3 \in W$.
Replacing s_1 by $s_1\,\alpha\, s_1^{/}$, where $s_1^{/} \in W$ and $\alpha \in \Gamma$ in the equation
$p(Q(s_1, s_2, s_3), n_2, n_3) = 0$, we get
$0 = p(Q(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3)$

$= p(Q(s_1, s_2, s_3)\,\alpha\, s_1^{/}, n_2, n_3) + p(s_1\,\alpha q\,(s_1^{/}, s_2, s_3), n_2, n_3)$
$= p(Q(s_1, s_2, s_n), n_2, n_3)\,\alpha\, s_1^{/} + Q(s_1, s_2, s_3)\alpha p(s_1^{/}, n_2, n_3) + p(s_1, n_2, n_3)\,\alpha q\,(s_1^{/}, s_2, s_3) + s_1\,\alpha p(q\,(s_1^{/}, s_2, s_3), n_2, n_3)$
$= Q(s_1, s_2, s_3)\alpha p(s_1^{/}, n_2, n_3)$
For every $s_1^{/}, s_1, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.
Replacing $s_1^{/}$ by $s_1^{/}\,\beta\, v$, where $v \in W$ and $\beta \in \Gamma$ in previous equation and using it again, we get
$0 = Q(s_1, s_2, s_3)\alpha p(s_1^{/}\,\beta\, v, n_2, n_3)$
$= Q(s_1, s_2, s_3)\alpha p(s_1^{/}, n_2, n_3)\,\beta v + Q(s_1, s_2, s_3)\alpha s_1^{/}\,\beta p(v, n_2, n_3)$
$= Q(s_1, s_2, s_3)\alpha\, s_1^{/}\,\beta p(v, n_2, n_3)$
for every $v, s_1^{/}, s_1, s_2, s_3, n_2, n_3 \in W$, $\alpha, \beta \in \Gamma$.
Hence by Lemma 1.16, we have
$p(v, n_2, n_3)\,\alpha\, Q(s_1, s_2, s_3) = 0$
for every $v, s_1, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.
Then (i) follows from (ii)
(iv) ⇒(i) by assumption we have
$P(Q(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3) = P(Q(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s_1^{/} + s_1\,\alpha\, p(q\,(s_1^{/}, s_2, s_3), n_2, n_3)$
for all $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.
And we also obtained
$P(Q(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3)$
$= P(Q(s_1, s_2, s_3)\,\alpha\, s_1^{/} + s_1\,\alpha q\,(s_1^{/}, s_2, s_3), n_2, n_3)$
$= P(Q(s_1, s_2, s_3)\,\alpha\, s_1^{/}, n_2, n_3) + P(s_1\,\alpha q\,(s_1^{/}, s_2, s_3), n_2, n_3)$
$= P(Q(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s_1^{/} + Q(s_1, s_2, s_3)\,\alpha p(s_1^{/}, n_2, n_3) + P(s_1, n_2, n_3)\,\alpha q\,(s_1^{/}, s_2, s_3) + s_1\,\alpha p(q\,(s_1^{/}, s_2, s_3), n_2, n_3)$
for every $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.
Comparing the above two expression of $P(Q(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3)$ we get
$Q(s_1, s_2, s_3)\,\alpha p(s_1^{/}, n_2, n_3) + P(s_1, n_2, n_3)\,\alpha q\,(s_1^{/}, s_2, s_3) = 0$ (2.1)
for every $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.
Also by assumption we have
$P(s_1, s_2, s_3)\,\alpha\, Q(n_1, n_2, n_3) = 0$ (2.2)
for all $s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha \in \Gamma$.
Replacing n_1 by $n_1\,\beta\, n_1^{/}$ in (2.2) and using it again, we get
$0 = P(s_1, s_2, s_3)\,\alpha\, Q(n_1\,\beta\, n_1^{/}, n_2, n_3)$
$= P(s_1, s_2, s_3)\,\alpha\, Q(n_1, n_2, n_3)\,\beta\, n_1^{/} + P(s_1, s_2, s_3)\,\alpha\, n_1\,\beta\, q\,(n_1^{/}, n_2, n_3)$
$= P(s_1, s_2, s_3)\,\alpha\, n_1\,\beta\, q\,(n_1^{/}, n_2, n_3)$
For every $s_1, s_2, s_3, n_1, n_1^{/}, n_2, n_3 \in W$ and $\alpha, \beta \in \Gamma$.
Thus it follows from Lemma 1.16 that
$P(s_1, s_2, s_3)\,\alpha\, q\,(n_1^{/}, n_2, n_3) = 0$
For every $s_1, s_2, s_3, n_1^{/}, n_2, n_3 \in W$ and $\alpha \in \Gamma$. (2.3)
Substituting equation (2.3) in the equation (2.1) yields
$Q(s_1, s_2, s_3)\,\alpha\, p(s_1^{/}, n_2, n_3) = 0$
For every $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.
Replacing $s_1^{/}$ by $s_1^{/}\,\beta\, v$, where $v \in W$ and $\beta \in \Gamma$ in previous equation and using it again, we get
$0 = Q(s_1, s_2, s_3)\,\alpha\, p(s_1^{/}\,\beta\, v, n_2, n_3)$
$= Q(s_1, s_2, s_3)\,\alpha p(s_1^{/}, n_2, n_3)\,\beta v + Q(s_1, s_2, s_3)\,\alpha\, s_1^{/}\,\beta p(v, n_2, n_3)$
$= Q(s_1, s_2, s_3)\,\alpha\, s_1^{/}\,\beta p(v, n_2, n_3)$
for every $v, s_1^{/}, s_1, s_2, s_3, n_2, n_3 \in W$ and $\alpha, \beta \in \Gamma$.
Hence by Lemma 1.16, we have
$p(v, n_2, n_3)\,\alpha\, Q(s_1, s_2, s_3) = 0$
for every $v, s_1, s_2, s_3, n_2, n_3 \in W$ and $\alpha \in \Gamma$
Hence by (ii), gives the result.

Theorem 2.4 :

Assume that W is a 2-torsion free semiprime Γ-near-ring and P, Q are two generalized 3-derivations of W with associated 3-derivations p and q of W respectively. If P and q are perpendicular and Q and p are perpendicular, then we have

(i) $p(q(s_1, s_2, s_3), n_2, n_3) = 0$ and
$P(Q(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3) = P(Q(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s_1^{/}$
for all $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$ and $\alpha \in \Gamma$.
(ii) $q(p(s_1, s_2, s_3), n_2, n_3) = 0$ and
$Q(P(s_1\,\alpha\, s_1^{/}, s_2, s_3), n_2, n_3) = Q(P(s_1, s_2, s_3), n_2, n_3)\,\alpha\, s_1^{/}$
for all $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.

Proof :

(i)since P and q are perpendicular , we have

$P(s_1, s_2, s_3)\, \alpha\, s\, \beta\, q(n_1, n_2, n_3) = 0$

for all $s, s_1, s_2, s_3, n_1, n_2, n_3 \in W$ and $\alpha, \beta \in \Gamma$.

Replacing s_1 by $s_1^{/}\, \delta\, s_1$ in above equation , by Lemma 2.2 , we have

$0 = P(s_1^{/}\, \delta\, s_1, s_2, s_3)\, \alpha\, s\, \beta\, q(n_1, n_2, n_3)$

$= P(s_1^{/}, s_2, s_3)\, \delta\, s_1\, \alpha\, s\, \beta\, q(n_1, n_2, n_3)$

$+ s_1^{/}\, \delta\, p(s_1, s_2, s_3)\, \alpha\, s\, \beta\, q(n_1, n_2, n_3)$

$= s_1^{/}\, \delta\, p(s_1, s_2, s_3)\, \alpha\, s\, \beta\, q(n_1, n_2, n_3)$

for all $s, s_1, s_1^{/}, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha, \beta, \delta \in \Gamma$.

Semiprimeness of W yields that

$p(s_1, s_2, s_3)\, \alpha\, s\, \beta\, q(n_1, n_2, n_3) = 0$

for every $s, s_1, s_2, s_3, n_1, n_2, n_3 \in W$,$\alpha, \beta \in \Gamma$.

Thus p and q are perpendicular

By Theorem 2.3 , we have

$p(q(s_1, s_2, s_3), n_2, n_3) = 0$

for every $s_1, s_2, s_3, n_2, n_3 \in W$. (2.4)

Since P and q are perpendicular and Q and p are perpendicular, we have

$P(s_1, s_2, s_3)\, \alpha\, q(n_1, n_2, n_3) = 0$ (2.5)

and

$Q(s_1, s_2, s_3)\, \alpha\, p(n_1, n_2, n_3) = 0$ (2.6)

for every $s_1, s_2, s_3, n_1, n_2, n_3 \in W$, $\alpha \in \Gamma$.

So we get

$P(Q(s_1\, \alpha\, s_1^{/}, s_2, s_3), n_2, n_3)$

$= P(Q(s_1, s_2, s_3)\, \alpha\, s_1^{/} + s_1\, \alpha q(s_1^{/}, s_2, s_3), n_2, n_3)$

$= P(Q(s_1, s_2, s_3)\, \alpha\, s_1^{/}, n_2, n_3) + P(s_1\, \alpha q(s_1^{/}, s_2, s_3), n_2, n_3)$

$= P(Q(s_1, s_2, s_3), n_2, n_3)\, \alpha\, s_1^{/} + Q(s_1, s_2, s_3)\, \alpha p(s_1^{/}, n_2, n_3) +$
$P(s_1, n_2, n_3)\, \alpha q(s_1^{/}, s_2, s_3) + s_1\, \alpha p(q(s_1^{/}, s_2, s_3), n_2, n_3)$

By using relation (2.4) , (2.5) and (2.6) in the last equation , we get

$P(Q(s_1\, \alpha\, s_1^{/}, s_2, s_3), n_2, n_3) = P(Q(s_1, s_2, s_3), n_2, n_3)\, \alpha\, s_1^{/}$

for all $s_1, s_1^{/}, s_2, s_3, n_2, n_3 \in W$, $\alpha \in \Gamma$.

(ii)The proof is the same way to the proof of (i)

References

[1] E. F. Adhab , 2016 . n-Additive Mappings in Prime and Semiprime Near-Rings , PHD. Thesis , Baghdad University , Iraq .

[2] I. N. Herstein , 1969 . Topics in Ring Theory , University of Chicago Press , Chicago .

[3] K.K. Dey , A. C. paul and I. S. Rakhimov , 2012 . On prime gamma-near-rings with generalized derivations , International Journal of Math. and Math. Sci. , doi: 10.1155/2012/625968 .

[4] K. K. Dey , A. C. Paul and I. S. Rakhimov , 2012 . Orthogonal Generalized Derivations in Semiprime Gamma Near-Rings , International Journal of Algebra, 6(23) : 1127-1134 .

[5] W. E. Barnes , 1966 . On the Γ-rings of Nabusawa ,Pacific J. Math. , 18 : 411-422 .

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على الحلقات المقتربة كاما شبه الاولية مع الاشتقاقات ـ 3 المعممة المتعامدة

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المستخلص:

في هذا البحث قدمنا مفهوم الاشتقاقات – 3 المعممة المتعامدة في الحلقات المقتربة كاما شبه الاولية وقدمنا العديد من الشروط الكافية والضرورية للاشتقاقات – 3 المعممة على الحلقات المقتربة كاما شبه الاولية لتصبح متعامدة .

الكلمات المفتاحية:

الحلقة المقتربة كاما شبه الاولية ، الاشتقاقات ـ3 ، الاشتقاقات ـ3 المعممة ، الاشتقاقات ـ3 المتعامدة ، الاشتقاقات ـ3 المعممة المتعامدة .